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R. KERRY ROWE, Ph.D., P.Eng.

List of Publications

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PUBLICATIONS

Refereed Journals - Full Papers

1. Rowe, R.K., Booker, J.R. and Balaam, N.P. (1978). "Application of the initial stress method to soil structure interaction," International Journal of Numerical Methods in Engineering, Vol. 12, No. 5, pp. 873-880.

2. Rowe, R.K. and Booker, J.R. (1979). "A method of analysis of horizontally embedded anchors in an elastic soil," International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 3, No. 2, pp. 187-203.

3. Rowe, R.K. and Booker, J.R. (1980). "The elastic response of multiple underream anchors," International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 4, No. 4, pp. 313-332.

4. Rowe, R.K. and Booker, J.R. (1981). "The elastic displacements of single and multiple underream anchors in a Gibson soil," Geotechnique, Vol. 31, No. 1, pp. 125-142.

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6. Rowe, R.K. and Booker, J.R. (1981). "The behaviour of footings on a non-homogeneous soil mass with a crust - Part II - circular footings," Canadian Geotechnical Journal, Vol. 18, No. 2, pp. 265-279.

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8. Rowe, R.K. (1982). "The determination of rock mass modulus variation with depth for weathered or jointed rock," Canadian Geotechnical Journal, Vol. 19, No. 1, pp. 29-43.

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10. Rowe, R.K. and Davis, E.H. (1982). "The behaviour of anchor plates in sand," Geotechnique, Vol. 32, No. 1, pp.25-41.

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12. Rowe, R.K., Lo, K.Y. and Kack, G.J. (1983). "A method of estimating surface settlement above tunnels constructed in soft ground," Canadian Geotechnical Journal, Vol. 20, No. 1, pp. 11-22.

13. Rowe, R.K. and Kack, G.J. (1983). "A theoretical examination of the settlements induced by tunnelling: Four case histories," Canadian Geotechnical Journal, Vol. 20, No. 2, pp. 299-314.

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16. Rowe, R.K. and Booker, J.R. (1984). "Deformation analysis for periodically layered soil," Journal of Geotechnical Engineering, ASCE, Vol. 110, GT2, pp. 217-230.

17. Rowe, R.K. (1984). "Reinforced embankments: analysis and design," Journal of Geotechnical Engineering, ASCE, Vol. 110, GT2, pp. 231-246.

18. Rowe, R.K., MacLean, M.D. and Barsvary, A.K. (1984). "The observed behaviour of a geotextile reinforced embankment constructed on peat," Canadian Geotechnical Journal, Vol. 21, No. 2, pp. 289-304.

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21. Rowe, R.K. and Booker, J.R. (1984). "The analysis of pollutant migration in a non-homogeneous soil," Geotechnique, Vol. 34, No. 4, pp. 601-612.

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24. Rowe, R.K. and Soderman, K.L. (1985). "An approximate method for estimating the stability of geotextile reinforced embankments," Canadian Geotechnical Journal, Vol. 22, No. 3, pp. 392-398.

25. Rowe, R.K. and Soderman, K.L. (1985). "Geotextile reinforcement of embankments on peat," International Journal for Geotextiles and Geomembranes, Vol. 2, No. 4, pp. 277-298.

26. Rowe, R.K. and Booker, J.R. (1985). "2D pollutant migration in soils of finite depth," Canadian Geotechnical Journal, Vol. 22, No. 4, pp. 429-436.

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30. Rowe, R.K. and Soderman, K.L. (1986). "Reinforced embankments on very poor foundations," International Journal of Geotextiles and Geomembranes, Vol. 4, No. 1, pp. 65-81.

31. Rowe, R.K. and Armitage, H.H. (1987). "Theoretical solutions for axial deformation of drilled shafts in rock," Canadian Geotechnical Journal, Vol. 24, No. 1, pp. 65-81.

32. Rowe, R.K. and Armitage, H.H. (1987). "A design method for drilled piers in soft rock," Canadian Geotechnical Journal, Vol. 24, No. 1, pp. 114-125.

33. Booker, J.R. and Rowe, R.K. (1987). "One dimensional advective-dispersive transport into a deep layer having a variable surface concentration," International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 11, No. 2, pp. 131-142.

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36. Rowe, R.K. (1983). "The behaviour of anchor plates in sand: Discussion," Geotechnique, Vol. 33, No. 1, p. 74.

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41. Rowe, R.K. and Poulos, H.G. (1979). "A method for predicting the effect of piles on slope behaviour," Proceedings of the 3rd International Conference on Numerical Methods in Geomechanics, Aachen, April 2-6, pp. 1073-1086.

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46. Rowe, R.K. and Booker, J.R. (1980). "The analysis of multiple underream anchors," Proceedings of the 3rd Australian and New Zealand Conference on Geomechanics, Wellington, May, Vol. 2, pp. 247-252.

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48. Pells, P.J.N. and Rowe, R.K. (1981). "A design method for rock socketed piles," Proceedings of the Symposium on Socketed Foundations on Weak Rock, Institution of Engineers, Australia, Brisbane, May, pp. 1-12.

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51. Rowe, R.K., Lo, K.Y. and Tham, L.G. (1982). "The analysis of tunnels and shafts in dense oil sands," Proceedings of the 4th International Conference on Numerical Methods in Geomechanics, Edmonton, May/June, pp. 587-596.

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54. Rowe, R.K. (1982). "Geotechnical aspects of tunnelling and pipe jacking," Ontario Concrete Pipe Association Seminar, Concrete Pipe '82, Toronto, October, pp. 1-28.

55. Folkes, D.J., Fisher, D.G. and Rowe, R.K. (1982). "Applications of geotextiles in artificial island construction," Seminar on Geotextiles, Calgary, November, pp. 1-42.

56. Lo, K.Y., Rowe, R.K. and Wai, R.S.C. (1983). "Thermal-mechanical response of a tunnel in limestone to an external heat source," Proceedings of the 7th Pan American Conference on Soil Mechanics and Foundation Engineering, Vancouver, pp. 567-580.

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63. Rowe, R.K. and Booker, J.R. (1985). "Pollutant transport through clayey soils and underlying aquifers," Proceedings of the 21st Congress of the International Association for Hydraulic Research, Melbourne, Vol. 1, pp. 159-164.

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67. Rowe, R.K. and Ho, S.K. (1986). "Determination of geotextile stress-strain characteristics using a wide strip test," Proceedings 3rd International Geotextile Conference, Vienna, pp. 885-890.

68. Quigley, R.M. and Rowe, R.K. (1986). "Leachate migration through clay below a domestic waste landfill, Sarnia, Ontario, Canada: chemical interpretation and modelling philosophies," Hazardous and Industrial Solid Waste Testing and Disposal: Sixth Volume, ASTM STP 933, pp. 93-103. (Proc. 5th Int'l. Symp. on Industrial and Hazardous Wastes, Alexandria, Egypt, June 1985.)

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71. Rowe, R.K. and Soderman, K.L. (1987). "Very soft soil stabilization using high strength geosynthetics: The role of Finite Element Analysis," Proceedings 1st Geosynthetics Research Institute Symposium, Drexel University, Philadelphia, October, 1987, pp. 58-87.

72. Rowe, R.K. and Booker, J.R. (1987). "New theoretical models for waste disposal sites with clay liners," Waste Geotechnics, Ed. A.S. Balasubramanian, Balkema (In Press).

73. Rowe, R.K. and Soderman, K.L. (1987). "Reinforcement of embankments on soils whose strength increases with depth," Proceedings of Geosynthetics '87, New Orleans, pp. 266-277.

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75. Rowe, R.K. and Armitage, H.H. (1987). "A new design method for drilled piers in soft rock: implications relating to three published case histories," Proceedings of 6th International Congress on Rock Mechanics, Montreal, pp. 497-502.

76. Rowe, R.K. and Booker, J.R. (1987). "Modelling of contaminant movement through fractured or jointed media with parallel fractures," Proceedings of 6th International Conference on Numerical Methods in Geomechanics, Innsbruck, April 1988 (To appear).

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Conference/Seminar/Symposium Proceedings

79. "Recent Developments in Geotechnical Instrumentation and Monitoring," published by the Canadian Geotechnical Society (SOS) & Tunnelling Association of Canada, March 1984 - R.K. Rowe - Editor.

80. "The Use of Geotextiles, Geogrids and Geomembranes in Engineering Practice," published by the Canadian Geotechnical Society, November 1984 - R.K. Rowe - Editor.

Books

81. Rowe, R.K. and Booker, J.R. (1987). "An Efficient Analysis of Pollutant Migration Through Soil," Chapter 2 in the Book "Numerical Methods in Transient and Coupled Systems" Ed. Lewis, Hinton, Bettess & Schrefler, John Wiley & Sons, pp. 13-42.

Scholarly Addresses

1. "A method for predicting the effect of piles on slope behaviour," at the 3rd International Conference on Numerical Methods in Geomechanics, Aachen, April, 1979.

2. "The analysis of inclined anchor plates," at the 3rd International Conference on Numerical Methods in Geomechanics, Aachen, April, 1979.

3. "An approach to soil-structure interaction analysis," Civil Seminar, U.W.O., April 1979.

4. "A theoretical study of pile/rock socket behavior," at the International Conference on Structural Foundations on Rock, Sydney, May 1980.

5. "The analysis of multiple underream anchors," at 3rd Australia & New Zealand Conference on Geomechanics, Wellington, May 1980.

6. "The design of socketed piers," at Cornell University, Ithaca, June 1980.

7. "One dimensional consolidation of soil exhibiting periodic layering," at International Symposium on Mechanical Behavior of Structured Media, Ottawa, May 1981.

8. "The analysis of tunnels and shafts in dense oil sands," at the 4th International Conference on Numerical Methods in Geomechanics, Edmonton, June 1982.

9. "The influence of non-homogeneity upon the performance of liquid storage tanks," at the 4th International Conference on Numerical Methods in Geomechanics, Edmonton, June 1982.

10. "The analysis of an embankment constructed on a geotextile," at the 2nd International Conference on Geotextiles, Las Vegas, August 1982.

11. "Geotechnical aspects of tunnelling and pipe jacking," at the Annual Meeting of the Ontario Concrete Pipe Association, Toronto, October, 1982.

12. "Recent advances in the prediction of subsidence due to soft ground tunnelling" at Sydney University, Sydney, Australia, June 1983.

13. "The determination of geotextile parameters appropriate to reinforced embankments" at Sydney University, Sydney, Australia, June 1983.

14. "The design of geotextile reinforced embankments" at Sydney University, Sydney, Australia, June 1983.

15. "The analysis of pollutant migration through soil" at Sydney University, Sydney, Australia, July 1983.

16. "Some aspects of the design of clay barriers," Seminar on the Design and Construction of Municipal and Industrial Waste Disposal Facilities, Toronto, June, 1984.

17. "A novel technique for the analysis of 1-D pollutant migration" at the International Conference on Numerical Methods for Transient and Coupled Problems, Venice, July, 1984.

18. "The design, analysis and observed behaviour of a reinforced test embankment" at the University of Waterloo, September, 1984.

19. "Geotextile reinforcement of embankments" at the Royal Military College, of Canada, Kingston, Ontario, September, 1984.

20. "The design of piles socketed into weak rock" at the Division of Building Research, National Research Council of Canada, Ottawa, Ontario, December, 1984.

21. "The analysis of multiple-contaminant migration through clayey soils" 5th International Conference on Numerical Methods in Geomechanics, Nagoya, April, 1985.

22. "Research on migration of pollutants into groundwater," Annual Meeting of the Civil and Mining Engineering Foundation, The University of Sydney, Sydney, May, 1985.

23. "Analysis of 2D pollutant migration in a layer of soil," 10th Canadian Congress on Applied Mechanics, London, Ontario, June, 1985.

24. "Geotextile reinforcement of embankments on soft foundations," at The University of Sydney, Sydney, Australia, July, 1985.

25. "Advantages and limitations of geotextile reinforcement of embankment," at The University of Newcastle, Newcastle, Australia, July, 1985.

26. "Applications for geotextiles in geotechnical engineering," at The University of Sydney, Sydney, Australia, July 1985.

27. "The influence of workmanship on the observed and calculated settlements induced by tunnelling," at XI International Conference on Soil Mechanics and Foundation Engineering, San Francisco, August 1985.

28. "The effect of soil model on calculated settlements due to tunnelling," at XI International Conference on Soil Mechanics and Foundation Engineering, San Francisco, August 1985.

29. "Pollutant transport through clayey soils and underlying aquifers," at 21st Congress of International Association for Hydraulic Research, Melbourne, Australia, August 1985.

30. "Theory and applications for geotextile reinforcement of embankments on soft foundations," at Workshop on Theory and Applications of Earth Reinforcing, University of Queensland, Brisbane, November 1985.

31. "Synthetics in earth reinforcement," Specialist Seminar, Reinforced Earth Company, Sydney, December 1985.

32. "Calculating contaminant migration in groundwater using a semi-analytic technique," at 2nd International Symposium on Numerical Models in Geomechanics, Ghent, Belgium, April 1986.

33. "Numerical modelling of reinforced embankments constructed on weak foundations," at 2nd International Symposium on Numerical Models in Geomechanics, Ghent, Belgium, April 1986.

34. "Determination of geotextile stress-strain characteristics using a wide strip test," at 3rd International Conference on Geotextiles, Vienna, Austria, April 1986.

35. "The design of geotextile reinforced embankments on poor foundations," at Virginia State University and Polytechnic Institute (VPI), Blacksburg, Virginia, USA, April 1986.

36. "Reinforced embankments on poor foundations," at Canadian Geotechnical Society SOS meeting, Toronto, September 1986.

37. "Modelling of contaminant migration through soil using program POLLUTE," Short Course on "Geotechnical Engineering for Waste-Disposal Projects," University of Texas at Austin, USA, November 1986.

38. "Reinforcement of embankments on soils whose strength increases with depth," at Geosynthetics '87, New Orleans, Feb. 1987.

39. "Geosynthetics in soil improvement: functions, properties and tests," at the University of Sydney, Sydney, Australia, May 1987.

40. "Reinforced embankments on soft cohesive deposits," at the University of Sydney, Sydney, Australia, May 1987.

41. "Reinforced embankments on organic deposits," at the University of Sydney, Sydney, Australia, May 1987.

42. "Reinforced slopes," at the University of Sydney, Sydney, Australia, May 1987.

43. "Contaminant migration - Recent advances in the state-of-the-art," at the University of Sydney, Sydney, Australia, May 1987.

44. "Use of finite element techniques in the analysis of reinforced soil walls," at NATO Advanced Research Workshop, Kingston, Ont., June 1987.

45. "Pollutant transport through barriers," at ASCE Specialty Conference on Geotechnical Aspects of Waste Disposal '87, Ann Arbor, Michigan, June 1987.

46. "A new design method for drilled piers in soft rock: Implications relating to three case histories," at 6th International Congress on Rock Mechanics, Montreal, August 1987.

47. "Contaminant migration in groundwater: The role of analysis in the design of barriers," The 11th Canadian Geotechnical Colloquium, at 40th Canadian Geotechnical Conference, Regina, October 1987.

48. "Stabilization of very soft soils using high strength geosynthetics: The role of Finite Element analyses," at the GRI Seminar on Very Soft Soil Stabilization Using High Strength Geosynthetics, Philadelphia, October 1987.

R.K. ROWE

Research Grants Related to Contaminant Transport
(awarded on the basis of peer review)

1982-1985 NSERC Strategic Grant $115,400
"Leachate Migration Through Clays: A Field Research Study"
(with R.M. Quigley, J.L. Sullivan and A. Margaritis)

1982-1987 NSERC Operating Grants R.K. Rowe
Current Grant $43,720/Year*

1987-1989 NSERC Strategic Grant $ 70,080
"Organics, Metallics and Bacteria On a Clay/Waste Interface" (with R.M. Quigley and A. Margaritis)

* The average NSERC Operating Grant in Civil Engineering for 1987-88 was $21,864.

Numerical Methods for Transient and Coupled Problems
Edited by R. W. Lewis, E. Hinton, P. Bettess, and B. A. Schrefler

Chapter 2

An Efficient Analysis of Pollutant Migration through Soil

R. K. Rowe and J. R. Booker

2.1 INTRODUCTION

The potential for contamination of groundwater is now a major consideration in the design and construction of waste disposal sites in many countries. Often the movement of contaminant from the disposal pit is controlled either by siting the landfill in a natural clayey soil or by constructing a compacted clay liner between the disposal pit and the surrounding soil. The selection of the type and thickness of the liner requires consideration of the expected concentrations at observation points beneath the liner, at the boundaries of the disposal site and possibly, at specific monitoring points outside the site. These contaminant migration analyses can be performed using time-marching finite element techniques [1], however, to obtain accurate results at both small and large times, this requires a relatively refined finite element mesh (to accommodate high concentration gradients at low times) and considerable computational effort.

Many soil deposits are horizontally layered and it is not really necessary to use the finite element method. In these cases, an alternative finite layer procedure proposed by Rowe and Booker [2–6] can be adopted for directly calculating the concentration of contaminants of specified locations and times. This approach, which will be described in this chapter, takes account of the fact that the concentration of contaminant within the disposal pit may decrease as mass is transported into the soil while also allowing for the possible presence of a more permeable underlying stratum (aquifer) beneath the clay liner. The technique to be adopted involves taking the Laplace and, where appropriate, Fourier transforms of the governing equations, finding an analytic solution in transform space and then numerically inverting the transforms to obtain the concentrations of contaminant at selected positions and times.

The solution will be developed initially for the case of 1D (vertical) advective-dispersive transport in layered soil but allowing for horizontal

transport in an underlying aquifer. The procedure will then be generalized to two and three dimensions.

The appliction of these techniques will be discussed and will be illustrated by a number of examples.

2.2 GOVERNING EQUATIONS

The transport of substances through a saturated clay can often be approximated by a Fickian-type law [7] having the form:

$$f_x = nv_x c - nD_{xx}\frac{\partial c}{\partial x} \tag{2.1a}$$

$$f_y = nv_y c - nD_{yy}\frac{\partial c}{\partial y} \tag{2.1b}$$

$$f_z = nv_z c - nD_{zz}\frac{\partial c}{\partial z} \tag{2.1c}$$

where

c is the concentration of the contaminant of interest at some point (x, y, z) at some time t

n is the porosity of the soil

f_x, f_y, f_z are the fluxes in the x, y, z Cartesian direction

v_x, v_y, v_z are the components of the seepage velocity in the x, y and z directions, and

D_{xx}, D_{yy}, D_{zz} are the coefficients of hydrodynamics dispersion (incorporating the effects of molecular diffusion and mechanical dispersion) in the x, y and z directions.

Consideration of mass balance gives:

$$\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} + n\frac{\partial c}{\partial t} + g = 0 \tag{2.2a}$$

where the quantity g takes account of the possibility of some of the contaminant being absorbed onto the clay skeleton. For equilibrium controlled ion exchange where the concentration of the exchange ion is relatively low, the absorption of this species may be approximated by a linear relationship of the form

$$g = \rho K\frac{\partial c}{\partial t} \tag{2.2b}$$

where

ρ is the dry density of the solid and

K is the distribution coefficient.

The distribution coefficient K may often be estimated from the results of a laboratory column test [8] or may be determined independently [9, 10]. It should of course be determined over a representative range of concentrations which reflect the likely field variation.

For a homogeneous layer in which the pore fluid velocity is uniform, equations (2.1) and (2.2) can be combined to give:

$$(n + \rho K)\frac{\partial c}{\partial t} = nD_{xx}\frac{\partial^2 c}{\partial x^2} + nD_{yy}\frac{\partial^2 c}{\partial y^2} + nD_{zz}\frac{\partial^2 c}{\partial z^2} - nv_x\frac{\partial c}{\partial x} - nv_y\frac{\partial c}{\partial y} - nv_z\frac{\partial c}{\partial z}. \tag{2.3a}$$

Equation (2.3a) governs 3D contaminant migration subject to the initial condition that

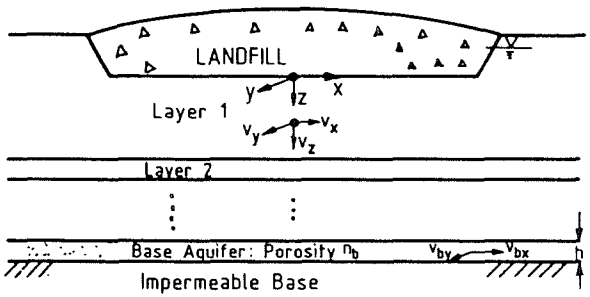
$$c = c_I \quad \text{at} \quad t = 0 \tag{2.3b}$$

where c_I is the initial (background) concentration.

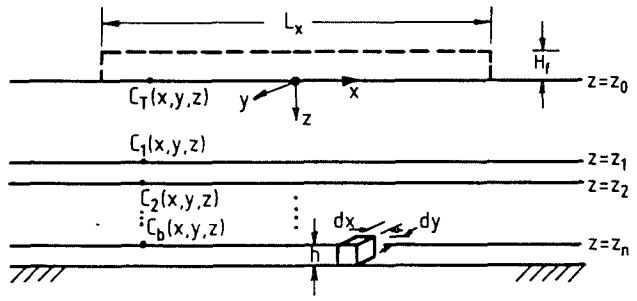
2.3 PROBLEM CONFIGURATION AND BOUNDARY CONDITIONS

A typical situation is shown schematically in Fig. 2.1. Supposing that at a depth $z = z_n$, the clayey deposit is underlain by a more permeable stratum (base aquifer) of thickness h and porosity n_b. If there is vertical advective transport into the aquifer then, strictly speaking, continuity requires that the base velocity in the aquifer should vary with horizontal position. However, if, as is often the case, the vertical velocity is small compared to the horizontal velocity in the aquifer then, as a first approximation, the base velocity may be assumed to be uniform and horizontal. If it is also assumed that the concentration in the aquifer is uniform across its thickness, then consideration of conservation of mass within a small element of the aquifer between $(x, x + \mathrm{d}x)$, $(y, y + \mathrm{d}y)$ gives:

$$n_b h\,\mathrm{d}x\,\mathrm{d}y\,c(x, y, z_n, t) = \mathrm{d}x\,\mathrm{d}y\int_0^t f_z(x, y, z_n, \tau)\,\mathrm{d}\tau - h\,\mathrm{d}y\int_0^t [f_x(x + \mathrm{d}x, y, z_n, \tau) - f_x(x, y, z_n, \tau)]\,\mathrm{d}\tau - h\,\mathrm{d}x\int_0^t [f_y(x, y + \mathrm{d}y, z_n, \tau) - f_y(x, y, z_n, \tau)]\,\mathrm{d}\tau \tag{2.4}$$



(a) Schematic of Landfill



(b) Idealization of Landfill

Figure 2.1 Problem under consideration.

which follows from the observation that the mass of contaminant in the element at time t is equal to the total mass transported into the element from the overlying clay less the net mass transported out of the element in the x and y Cartesian directions. Assuming again that mass transport is governed by Fick's Law, dividing throughout by the pore volume $n_b h\,dx\,dy$, and taking the limit as dx and dy tend to zero then gives:

$$c(x,y,z_n t) = \int_0^t \left[\frac{f_z(x,y,z_n,\tau)}{n_b h} - \frac{v_{bx}}{n_b}\frac{\partial c(x,y,z_n,\tau)}{\partial x} - \frac{v_{by}}{n_b}\frac{\partial c(x,y,z_n,\tau)}{\partial y} \right.$$

$$\left. + D_{Hx}\frac{\partial^2 c(x,y,z_n,\tau)}{\partial x^2} + D_{Hy}\frac{\partial^2 c(x,y,z_n,\tau)}{\partial y^2} \right] d\tau \qquad (2.5a)$$

where

n_b is the base aquifer porosity

h is the thickness of the base aquifer

v_{bx}, v_{by} are the components of the superficial velocity in the x and y directions in the base aquifer (The superficial velocity is the seepage velocity, multiplied by the porosity.)

D_{Hx}, D_{Hy} are the coefficients of hydrodynamic dispersion in the x and y directions in the base aquifer.

Adopting the following notation

$c_b = c(x, y, z_n, \tau)$ is the concentration in the aquifer at (x, y, z_n) at time τ;

$f_{bz} = f_z(x, y, z_n, \tau)$ is the flux into the aquifer at (x, y, z_n) at time τ;

$f_{bx} = f_x(x, y, z_n, \tau)$ is the flux in the x direction in the base aquifer at time τ; and

$f_{by} = f_y(x, y, z_n, \tau)$ is the flux in the y direction in the base aquifer at time τ.

Equation (2.5a) can be rewritten as

$$c_b = \int_0^t \left(\frac{f_{bz}}{n_b h} - \frac{v_{bx}}{n_b}\frac{\partial c_b}{\partial x} - \frac{v_{by}}{n_b}\frac{\partial c_b}{\partial y} + D_{Hx}\frac{\partial^2 c_b}{\partial x^2} + D_{Hy}\frac{\partial^2 c_b}{\partial y^2} \right) d\tau \qquad (2.5b)$$

Thus the presence of a base aquifer can be modelled as a boundary condition by invoking equation (2.5). Of course when analysing a layered deposit for 2D or 3D conditions an aquifer could also be modelled as an additional layer in the deposit rather than as a boundary condition to the clayey deposit.

In many practical situations, the mass of solute in the landfill (or lagoon) will be limited and the mass will reduce with time as pollutant is transported from the landfill. The simplest such case is where the landfill is filled relatively quickly having a concentration c_0 and height of fluid H_f at completion. The height of fluid H_f represents the volume of leachate (which depends on the porosity of the landfill and the location of the watertable) divided by the average plan area of the landfill. The initial concentration c_0 and height of leachate H_f can be estimated for a given landfill. Assuming that the concentration in the landfill is spatially homogeneous but may vary with time as mass is transported into the clay, conservation of mass requires that:

$$AH_f c_{LF}(t) = AH_f c_0 - \int_0^t \left(\iint_A f_T\, dA \right) d\tau. \qquad (2.6a)$$

where

A is the plan area of the landfill;

H_f is the equivalent height of leachate (fluid);

$c_{LF}(t)$ is the concentration of the contaminant within the landfill at time t;

c_0 is the initial concentration of contaminant within the landfill (i.e. at $t = 0$); and

$f_T = f_z(x, y, z_0, \tau)$ is the flux entering the soil from the landfill at the point (x, y, z_0) at time τ and may be calculated from (2.1).

Equation (2.6a) simply says that the mass of contaminant within the landfill at time t is equal to the original mass of contaminant within the landfill (at $t = 0$) less the mass of contaminant transported into the soil over the area of the landfill between time $\tau = 0$ and $\tau = t$. Dividing (2.6a) throughout by AH_f then gives an expression for the concentration c_{LF} within the landfill at time t

$$c_{LF}(t) = c_0 - \frac{1}{AH_f} \int_0^t \left(\iint_A f_T \, dA \right) d\tau \tag{2.6b}$$

Clearly, the limit as the volume of fluid in the landfill tends to infinity ($H_f \to \infty$) corresponds to a constant surface concentration.

2.4 1D SOLUTION

The simplest case which can be considered is that of one-dimensional contaminant transport in the clayey deposit, parallel to the z direction as shown in Fig. 2.1. It is assumed that this deposit is divided into a number of layers by node planes $z = z_0, z_1, \ldots, z_n$ and that each layer, $(z_{k-1} \leqslant z < z_k)$ may be considered homogeneous. Thus the vertical flux per unit area per unit time at a point in layer k is given by (2.1) viz.:

$$f_z = nv_z c - nD_{zz} \frac{\partial c}{\partial z}. \tag{2.1c}$$

For 1D mass transport in the clayey deposit, the equation governing pollutant migration (2.3) reduces to

$$(n + \rho K) \frac{\partial c}{\partial t} = nD_{zz} \frac{\partial^2 c}{\partial z^2} - nv_z \frac{\partial c}{\partial z}. \tag{2.7a}$$

Subject to the initial condition

$$c = c_I(z_{k-1} \leqslant z \leqslant z_k) \quad \text{at } t = 0 \tag{2.7b}$$

where the quantities n, v_z, D_{zz}, ρ, K and c_I assume constant values appropriate for the layer k under consideration.

Equations (2.1c) and (2.7) can be simplified by introducing the Laplace Transform

$$(\bar{c}, \bar{f}_z) = \int_0^\infty (c, f_z) \, e^{-st} \, dt \tag{2.8}$$

yielding

$$\bar{f}_z = nv_z \bar{c} - nD_{zz} \frac{\partial \bar{c}}{\partial z} \tag{2.9}$$

$$(n + \rho K)(s\bar{c} - c_I) = nD_{zz} \frac{\partial^2 \bar{c}}{\partial z^2} - nv_z \frac{\partial \bar{c}}{\partial z}. \tag{2.10}$$

Equation (2.10) has a solution of the form

$$\bar{c} = E + A \, e^{\alpha z} + B \, e^{\beta z} \tag{2.11a}$$

where $m = \alpha, \beta$ are the roots of the equation

$$nD_{zz}m^2 - nv_z m - (n + \rho K)s = 0 \tag{2.11b}$$

$$\begin{matrix}\alpha \\ \beta\end{matrix} = \frac{v_z}{2D_{zz}} \pm \left(\frac{v_z^2}{4D_{zz}^2} + \frac{s(n + \rho K)}{nD_{zz}} \right)^{1/2} \tag{2.11c}$$

and

$$E = c_I/s = -\frac{(n + \rho K)c_I}{nD_{zz}\alpha\beta}. \tag{2.11d}$$

Evaluating equations (2.11) in terms of the concentrations at the nodal planes z_j, z_k (where $j = k - 1$) yields the interpolation formulae

$$\bar{c} = (\bar{c}_j - E) \left\{ \frac{e^{\alpha(z - z_k)} - e^{\beta(z - z_k)}}{e^{\alpha(z_j - z_k)} - e^{\beta(z_j - z_k)}} \right\} + (\bar{c}_k - E) \left\{ \frac{e^{\alpha(z - z_j)} - e^{\beta(z - z_j)}}{e^{\alpha(z_k - z_j)} - e^{\beta(z_k - z_j)}} \right\} + E \tag{2.12}$$

and thus

$$\frac{\bar{f}_z}{nD_{zz}} = (\bar{c}_j - E) \left\{ \frac{\beta \, e^{\alpha(z - z_k)} - \alpha \, e^{\beta(z - z_k)}}{e^{\alpha(z_j - z_k)} - e^{\beta(z_j - z_k)}} \right\} + (\bar{c}_k - E) \left\{ \frac{\beta \, e^{\alpha(z - z_j)} - \alpha \, e^{\beta(z - z_j)}}{e^{\alpha(z_k - z_j)} - e^{\beta(z_k - z_j)}} \right\} + (\alpha + \beta)E. \tag{2.13}$$

If we denote the depth of the sublayer by $\lambda = z_k - z_j$ this then gives the layer matrix relating nodal concentrations and nodal fluxes

$$\begin{bmatrix} \bar{f}_{zj} \\ -\bar{f}_{zk} \end{bmatrix} = nD_{zz} \begin{bmatrix} \dfrac{\beta\, e^{-\alpha\lambda} - \alpha\, e^{-\beta\lambda}}{e^{-\alpha\lambda} - e^{-\beta\lambda}} & \dfrac{(\beta-\alpha)}{e^{\alpha\lambda} - e^{\beta\lambda}} \\ \dfrac{-(\beta-\alpha)}{e^{-\alpha\lambda} - e^{-\beta\lambda}} & \dfrac{-(\beta\, e^{\alpha\lambda} - \alpha\, e^{\beta\lambda})}{e^{\alpha\lambda} - e^{\beta\lambda}} \end{bmatrix} \begin{bmatrix} \bar{c}_j - E \\ \bar{c}_k - E \end{bmatrix}$$

$$+ (\alpha+\beta)EnD_{zz} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{2.14a}$$

which may be written

$$\begin{bmatrix} \bar{f}_{zj} \\ -\bar{f}_{zk} \end{bmatrix} = \begin{bmatrix} Q_k & R_k \\ S_{zk} & T_k \end{bmatrix} \begin{bmatrix} \bar{c}_j - E \\ \bar{c}_k - E \end{bmatrix} + (\alpha+\beta)EnD_{zz} \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \tag{2.14b}$$

Rearranging then gives:

$$\begin{bmatrix} \bar{f}_{zj} \\ -\bar{f}_{zk} \end{bmatrix} = \begin{bmatrix} Q_k & R_k \\ S_k & T_k \end{bmatrix} \begin{bmatrix} \bar{c}_j \\ \bar{c}_k \end{bmatrix} - \begin{bmatrix} U_k \\ V_k \end{bmatrix} \tag{2.14c}$$

where

$$U_k = [Q_k + R_k - (\alpha+\beta)nD_{zz}]E$$
$$V_k = [S_k + T_k + (\alpha+\beta)nD_{zz}]E$$

and where α, β, E are defined by (2.11c) and (2.11d).

Noting that both the flux f_z and concentrations c must be continuous, the layer matrices defined by equation (2.14) may be assembled for each layer in the deposit to give

$$\begin{bmatrix} Q_1 & R_1 & & & & & \\ S_1 & T_1+Q_2 & R_2 & & & & \\ & S_2 & T_2+Q_3 & R_3 & & & \\ & & & \vdots & & & \\ & & & S_{n-1} & T_{n-1}+Q_n & R_n & \\ & & & & S_n & T_n & \end{bmatrix} \begin{bmatrix} \bar{c}_T \\ \bar{c}_1 \\ \bar{c}_2 \\ \vdots \\ \bar{c}_{n-1} \\ \bar{c}_b \end{bmatrix} = \begin{bmatrix} \bar{f}_T + U_1 \\ V_1 + U_2 \\ \\ \vdots \\ V_{n-1} + U_n \\ -\bar{f}_b + V_n \end{bmatrix} \tag{2.15}$$

where

$\bar{c}_T, \bar{f}_T$ are the values of concentration and flux at the top of the deposit; and

$\bar{c}_b = \bar{c}_n$, $\bar{f}_b = \bar{f}_n$ are the values of concentration and flux at the base of the deposit.

We now wish to solve this equation subject to the appropriate boundary conditions.

Equation (2.15) describes vertical contaminant migration in a clayer deposit for 1D conditions. If this deposit is underlain by a far more permeable stratum with ground-water flow in the horizontal direction at a superficial velocity v_b, then solute will be transported away from the landfill at a rate dependent on the velocity, porosity and geometric dimensions of this layer. The concentration in this base aquifer will only tend to zero when the base velocity is sufficiently large to remove the discharged leachate. However, in many cases the velocity will be relatively small and there will be a change in concentration with time in this stratum. To provide a means of estimating this concentration in a one dimensional analysis, it will be assumed that the concentration $c_b(t)$ in the base aquifer does not vary with vertical or horizontal position and that solute transport in this layer is only by horizontal advection. Thus for a landfill of length L (where the length is the dimension of the landfill parallel to the base velocity v_b), the general equation (2.5) governing the concentration in the base reduces to

$$c_b = \int_0^t \left(\frac{f_{bz}}{n_b h} - \frac{v_b}{n_b} \frac{c_b}{L} \right) d\tau \tag{2.16}$$

where c_b represents an average concentration in the aquifer beneath the landfill.

It is implicitly assumed here that the permeable stratum is confined by a lower impermeable boundary and so the volume of water in this permeable stratum, beneath the landfill, depends on its thickness, h. Thus, the assumption that c_b is independent of position is likely to be most appropriate when h is no more than a few metres. This approximation is considered to be adequate for many practical situations; the more general two-dimensional case where the concentration varies with lateral position beneath the landfill will be considered in the next section.

Taking the Laplace transform of (2.16) gives

$$\bar{c}_b = \frac{\bar{f}_b}{s n_b h} - \frac{\bar{v}_b \bar{c}_b}{s n_b L} \tag{2.17}$$

This can be rewritten in the form

$$\bar{f}_b = \Omega \bar{c}_b \tag{2.18a}$$

where

$$\Omega = h[n_b s + v_b/L]. \tag{2.18b}$$

Substituting (2.18a) into the last equation of (2.15) then gives

$$S_n \bar{c}_{n-1} + (T_n + \Omega)\bar{c}_b = V_n \tag{2.19}$$

which now replaces the last equation of (2.15).

Now considering the boundary condition at the top nodal plane $z = z_0$, the top concentrate c_T is equal to that in the landfill i.e.:

$$c_T = c_{LF} \qquad t > 0 \tag{2.20}$$

and for the 1D case, the equation for the concentrations in the landfill (2.6b) reduces to

$$c_{LF}(t) = c_0 - \frac{1}{H_f}\int_0^t f_T \, d\tau. \tag{2.21a}$$

Taking the Laplace transform, this becomes

$$\bar{c}_{LF} = \frac{\bar{c}_0}{s} - \frac{\bar{f}_T}{sH_f}. \tag{2.22}$$

Combining (2.20) and (2.22) and the first equation of (2.15) then gives:

$$\bar{c}_{LF} = \frac{\bar{c}_0}{s} - \frac{\bar{f}_T}{sH_f} \tag{2.22}$$

Combining (2.20) and (2.22) and the first equation of (2.15) then gives:

$$(Q_1 + sH_f)c_{LF} + R_1 c_1 = c_0 H_f + U_1. \tag{2.23}$$

Replacing the first and last equation of (2.15) by (2.23) and (2.19) then gives the complete set of equations.

$$\begin{bmatrix} Q_1 + sH_f & R_1 & & & & & & \\ S_1 & T_1 + Q_2 & R_2 & 0 & & & & \\ 0 & S_2 & T_2 + Q_3 & R_3 & & & & \\ & & & \cdot & S_{n-1} & T_n + Q_n & R_n & \\ & & & & 0 & S_n & T_n + \Omega \end{bmatrix} \begin{bmatrix} \bar{c}_{LF} \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_{n-1} \\ \bar{c}_b \end{bmatrix} = \begin{bmatrix} c_0 H_f + U_1 \\ V_1 + U_2 \\ V_2 + U_3 \\ V_{n-1} + U_n \\ V_n \end{bmatrix}$$

where c denotes the concentration in layer plane $z = z_j$; Q_k, R_k, S_k, T_k, U_k and V_k are defined by equation (2.14); and Ω is defined by (2.18b). Equation (2.24) can now be solved giving $\bar{c}_j$ at each node plane. The Laplace transform can then be inverted using a very efficient scheme proposed by Talbot [12]. Using this approach an accuracy of order 10^{-6} and 10^{-10} can typically be achieved using 11 and 18 sample points respectively. For many practical problems an accuracy of 10^{-6} is more than adequate. The theory described above has been coded in program POLLUTE [11] and can be run on both mini and micro computers.

2.5 2D SOLUTION

The procedure described in the previous section can be readily generalized for contaminant transport in both the x and z directions. Again, it is assumed that the deposit is divided into a number of layers with node planes $z = z_0, z_1, \ldots z_n$

and that each layer $k(z_{k-1}, \leqslant z \leqslant z_k)$ may be considered as homogeneous. Thus from equation (2.1) the fluxes f_x, f_z transported in the x and y directions within layers k are given by

$$f_x = nv_x c - nD_{xx}\frac{\partial c}{\partial x} \tag{2.1a}$$

$$f_z = nv_z c - nD_{zz}\frac{\partial c}{\partial z}. \tag{2.1c}$$

For 2D transport in the soil, the equation governing contaminant migration (2.3) reduces to

$$(n + \rho K)\frac{\partial c}{\partial t} = nD_{xx}\frac{\partial^2 c}{\partial x^2} + nD_{zz}\frac{\partial^2 c}{\partial z^2} - nv_x\frac{\partial c}{\partial x} - nv_z\frac{\partial c}{\partial z} \tag{2.25a}$$

and

$$c = c_I(z_{k-1} \leqslant z \leqslant z_k, \; -\infty \leqslant x \leqslant \infty) \quad \text{at } t = 0 \tag{2.25b}$$

where the quantities n, v_x, v_z, D_{xx}, D_{zz}, ρ, K and c_I are constants appropriate to the layer k under consideration.

Equation (2.1a), (2.1c), (2.25a) and (2.25b) can be simplified by the introduction of a Laplace transform:

$$(\bar{c}, \bar{f}_x, \bar{f}_z) = \int_0^\infty (c, f_x, f_z)\, e^{-st}\, dt \tag{2.26}$$

and a Fourier transform

$$(C, F_x, F_z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} (c, f_x, f_z)\, e^{-i\xi x}\, dx \tag{2.27}$$

yielding

$$\bar{F}_x = nv_x\bar{C} - nD_{xx}i\xi\bar{C} \tag{2.28a}$$

$$\bar{F}_z = nv_z\bar{C} - nD_{zz}\frac{\partial\bar{C}}{\partial z} \tag{2.28b}$$

and

$$(n + \rho K)(s\bar{C} - C_I) = -\xi^2 nD_{xx}\bar{C} + nD_{zz}\frac{\partial^2\bar{C}}{\partial z^2} - i\xi nv_x\bar{C} - nv_z\frac{\partial\bar{C}}{\partial z}. \tag{2.29}$$

This equation has the solution

$$\bar{C} = E + A\, e^{\alpha z} + B\, e^{\beta z} \tag{2.30a}$$

where $m = \alpha, \beta$ are the roots of the equation:

$$nD_{zz}m^2 - nv_z m - [i\xi nv_x + \xi^2 nD_{xx} + (n + \rho K)s] = 0 \tag{2.30b}$$

and

$$E = \frac{(n+\rho K)C_I}{\xi^2 n D_{xx} + i\xi n v_x + (n+\rho K)s} = -\frac{(n+\rho K)C_I}{nD_{zz}\alpha\beta}. \tag{2.30c}$$

Evaluating equation (2.30) in terms of concentration of the nodal planes z_j, $z_k (j = k-1)$ yields the interpolation formulae:

$$\bar{C} = (\bar{C}_j - E)\left\{\frac{e^{\alpha(z-z_k)} - e^{\beta(z-z_k)}}{e^{\alpha(z_j-z_k)} - e^{\beta(z_j-z_k)}}\right\} + (\bar{C}_k - E)\left\{\frac{e^{\alpha(z-z_j)} - e^{\beta(z-z_j)}}{e^{\alpha(z_k-z_j)} - e^{\beta(z_k-z_j)}}\right\} + E \tag{2.31}$$

and thus

$$\frac{\bar{F}_z}{nD_{zz}} = (\bar{C}_j - E)\left\{\frac{\beta\, e^{\alpha(z-z_k)} - \alpha\, e^{\beta(z-z_k)}}{e^{\alpha(z_j-z_k)} - e^{\beta(z_j-z_k)}}\right\} + (\bar{C}_k - E)\left\{\frac{\beta\, e^{\alpha(z-z_j)} - \alpha\, e^{\beta(z-z_j)}}{e^{\alpha(z_k-z_j)} - e^{\beta(z_k-z_j)}}\right\} + (\alpha+\beta)E. \tag{2.32}$$

Notice that (2.31) and (2.32) have precisely the same form as (2.12) and (2.13) since only the definition of α, β and E differ. Consequently, the layer matrix also takes the same form as (14a) and can be written as

$$\begin{bmatrix} \bar{F}_{zj} \\ -\bar{F}_{zk} \end{bmatrix} = \begin{bmatrix} Q_k & R_k \\ S_k & T_k \end{bmatrix} \begin{bmatrix} \bar{C}_j \\ \bar{C}_k \end{bmatrix} - \begin{bmatrix} U_k \\ V_k \end{bmatrix} \tag{2.33}$$

where Q_k, R_k, S_k, T_k, U_k and V_k are as defined in (14) but where in this case α, β, E are defined by (2.30b) and (2.30c).

Noting again that the flux $\bar{F}_z$ and concentrations $\bar{C}$ must be continuous the layer matrices defined by (2.33) may be assembled for each layer k in the deposit to give

$$\begin{bmatrix} Q_1 & R_1 & & & & & \\ S_1 & T_1+Q_2 & R_2 & & & & \\ & S_2 & T_2+Q_3 & R_3 & & & \\ & & & S_{n-1} & T_{n-1}+Q_n & R_n & \\ & & & & S_n & T_n & \end{bmatrix} \begin{bmatrix} \bar{C}_T \\ \bar{C}_1 \\ \bar{C}_2 \\ \bar{C}_{n-1} \\ \bar{C}_b \end{bmatrix} = \begin{bmatrix} \bar{F}_T + U_1 \\ V_1 + U_2 \\ \\ V_{n-1} + U_n \\ -\bar{F}_b + V_n \end{bmatrix} \tag{2.34}$$

where $\bar{F}_T$ and $\bar{F}_b$ are the transformed fluxes at the top ($z = z_0$) and bottom ($z = z_n$) nodal plane respectively and $\bar{C}_T$ and $\bar{C}_b$ are the corresponding transforms of the concentrations at these points. It should be noted that this matrix will not usually be symmetric but it is tridiagonal and hence solution of these equations is computationally trivial.

Equation (2.34) must be solved subject to the appropriate boundary conditions. Considering the lower boundary first, suppose that the clay layer is

underlain by a more permeable stratum of thickness h and porosity n_b. Assuming that the concentration is uniform across the thickness h but may vary with position x, the general equation governing the concentration in the base aquifer (2.5) reduces to

$$c_b = \int_0^t \left(\frac{f_{bz}}{n_b h} - \frac{v_{bx}}{n_b}\frac{\partial c_b}{\partial x} + D_{Hx}\frac{\partial^2 c_b}{\partial x^2}\right) d\tau \tag{2.35}$$

If (2.35) is transformed using (2.26) and (2.27) it is found that:

$$\bar{F}_b = \Omega \bar{C}_b \tag{2.36a}$$

where

$$\Omega = h[n_b s + i\xi v_b + n_b D_{Hx}\xi^2]. \tag{2.36b}$$

Thus substituting (2.36a) into the last equation of (2.34) gives

$$S_n \bar{C}_{n-1} + (T_n + \Omega)\bar{C}_b = V_n. \tag{2.37}$$

Replacing the last equation of (2.34) by (2.37) allows the horizontal advective–dispersive transport in the aquifer to be modelled as a boundary condition. Clearly, the same result could also be achieved by modelling the aquifer as a layer of the deposit. If this aquifer layer is underlain by a no-flux boundary, this can then be simulated by setting $-F_b = 0$ in the last equation of (2.34) which now becomes.

$$S_n \bar{C}_{n-1} + T_n \bar{C}_b = V_n \tag{2.38}$$

The boundary condition at the upper boundary can be developed for the case where there is a finite mass of contaminant within the landfill of length L (see Fig. 2.1). Assuming that the concentration of contaminant within this landfill is spatially homogeneous but may vary with time as mass is transported into the underlying soil, the two dimensional version of (2.6) can be written

$$c_{LF}(t) = c_0 - \frac{1}{LH_f}\int_0^t \int_{-L/2}^{L/2} f_T \, dx \, dt \tag{2.39}$$

and so the concentration on the nodal plane $z = z_0$ is given by

$$\begin{aligned} c_T &= c(x, z=z_0, t) = c_{LF}(t) \qquad -L/2 \leqslant x \leqslant L/2 \\ c_T &= c(x, z=z_0, t) = 0 \qquad\quad |x| > L/2. \end{aligned} \tag{2.40}$$

Taking the Laplace and Fourier transforms of (2.40) then gives

$$\bar{C}_T = \frac{1}{2\pi}\int_{-L/2}^{L/2} \bar{C}_{LF}\, e^{-i\xi x}\, dx = \frac{L}{2\pi}\bar{C}_{LF}\frac{\sin(\xi L/2)}{(\xi L/2)} \tag{2.41}$$

where $\bar{C}_{LF}$ is as yet unknown.

To determine $\bar{C}_{LF}$ let us firstly define, $\bar{\chi}$ such that

$$\bar{\chi} = \bar{F}_T \tag{2.42}$$

for the reference case where $\bar{F}_T$ is determined by solving (2.34) (with the appropriate base boundary condition), subject to the condition that $\bar{C}_{LF} = 1$ and hence, from (2.41),

$$\bar{C}_T = \frac{L}{2\pi} \frac{\sin(\xi L/2)}{(\xi L/2)}. \tag{2.43}$$

It follows that in general

$$\bar{F}_T = \bar{\chi} \bar{C}_{LF}. \tag{2.44}$$

Taking the Laplace transform of (2.39) gives

$$\bar{c}_{LF} = \frac{c_0}{s} - \frac{1}{sLH_f} \int_{-L/2}^{L/2} \bar{f}_T \, dx \tag{2.45}$$

Now from the Fourier inversion theorem

$$\bar{f}_T = \int_{-\infty}^{\infty} \bar{F}_T \, e^{i\xi x} \, d\xi \tag{2.46}$$

and hence

$$\left.\begin{aligned} \int_{-L/2}^{L/2} \bar{f}_T \, dx &= \int_{-\infty}^{\infty} \int_{-L/2}^{L/2} \bar{F}_T \, e^{i\xi x} \, dx \, d\xi \\ &= \int_{-\infty}^{\infty} \frac{L \sin(\xi L/2)}{(\xi L/2)} \bar{F}_T \, d\xi \end{aligned}\right\} \tag{2.47}$$

substituting (2.44) and (2.47) into (2.45) then gives

$$\bar{c}_{LF} = \frac{c_0}{s} - \frac{1}{sLH_f} \int_{-\infty}^{\infty} \frac{L \sin(\xi L/2)}{(\xi L/2)} \bar{C}_{LF} \bar{\chi} \, d\xi \tag{2.48}$$

which upon solving for $\bar{c}_{LF}$ gives

$$\bar{c}_{LF} = \frac{LH_f c_0}{sLH_f + \Lambda} \tag{2.49a}$$

where

$$\Lambda = \int_{-\infty}^{\infty} \frac{2 \sin(\xi L/2)}{\xi} \bar{\chi} \, dz. \tag{2.49b}$$

The reference concentrations $\bar{c}_{rj}$ at the nodal planes are obtained for the reference condition where $\bar{c}_{LF} = 1$ by invoking

$$\bar{c}_{rj} = \int_{-\infty}^{\infty} \bar{C}_j \, e^{i\xi x} \, d\xi \qquad (1 \leqslant j \leqslant n) \tag{2.50}$$

and by performing the integration using numerical quadrature. The transform coefficients $\bar{C}_j$ in (2.50) are obtained by solving (2.34) subject to the appropriate base boundary condition (2.37) and (2.38) and the top boundary condition (2.43). At the same time the quantity Λ can be determined from (2.42) and (2.49b). The concentration $\bar{c}_{LF}$ can then be determined from (2.49a) and hence the concentrations of the nodal planes are given by

$$\bar{c}_j = \bar{c}_{LF} \cdot \bar{c}_{rj}. \tag{2.51}$$

It then only remains to invert the Laplace transform.

The theory described above for 2D conditions has been coded in program MIGRATE [13]. The major computational effort involved is associated with the numerical inversion of the Fourier and Laplace transforms for the locations and times of interest. The Fourier transform can be efficiently inverted using 20 point Gauss quadrature. The width and number of integration subintervals which are needed to achieve a reasonable accuracy (say 0.1%) depends somewhat on the geometry and properties of the problem under consideration. These parameters can be determined from a few trial calculations for a representative point and time of interest. (Similarly, it should be noted that numerical experiments are also required to determine an appropriate finite element mesh and time integration procedure if alternative finite element or finite difference codes are used.) The Laplace transform can again be inverted using Talbot's algorithm.

2.6 3D SOLUTIONS

The governing equations for the 3D case are given by (2.1) and (2.3). The development of the solution for this case follows along the same lines as that for the 2D case described in the previous section except that here the Fourier transform take the form

$$(C, F_x, F_y, F_z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (c, f_x, f_y, f_z) \, e^{-i\xi x - i\eta y} \, dx \, dy. \tag{2.52}$$

Thus taking the Laplace and Fourier Transform of (2.3) gives

$$\begin{aligned} (n + \rho K)(s\bar{C} - C_I) = &- \xi^2 n D_{xx} \bar{C} - \eta^2 n D_{yy} \bar{C} + n D_{zz} \frac{\partial^2 \bar{C}}{\partial z^2} \\ &- i\xi n v_x \bar{C} - i\eta n v_y \bar{C} - n v_z \frac{\partial \bar{C}}{\partial z} \end{aligned} \tag{2.53}$$

for any layer k ($z_{k-1} \leqslant z \leqslant z_k$).

This also has a solution of the form

$$\bar{C} = E + A \, e^{\alpha z} + \beta \, e^{\beta z} \tag{2.54a}$$

where $m = \alpha, \beta$ are the roots of the equation

$$n D_{zz} m^2 - n v_z m - [i\xi n v_x + i\eta n v_y + \xi^2 n D_{xx} + \eta^2 n D_{yy} + (n + \rho K)s] = 0 \tag{2.54b}$$

and

$$E = \frac{-(n+\rho K)c_I}{nD_{zz}\alpha\beta}. \tag{2.54c}$$

The layer matrices take precisely the same form as that given by (2.14) and (2.33) but where α, β, E are defined by (2.54b) and (2.54c) above. The layer matrices may then be assembled for each layer in the deposit to give (2.34) which again must be solved subject to the appropriate boundary conditions in a similar manner to that described for the 2D case.

Considering horizontal flow in a base aquifer of thickness h and porosity n_b, the equation governing the variation in concentration (within the aquifer) with lateral position and time is given by (2.5b).

Thus taking the Fourier and Laplace transforms of (2.5b) it is found that

$$\bar{F}_b = \Omega \bar{C}_b \tag{2.55a}$$

where

$$\Omega = h[n_b s + i\xi v_{bx} + i\eta v_{by} + n_b D_{Hx}\xi^2 + n_b D_{Hy}\eta^2] \tag{2.55b}$$

which can then be substituted into the last equation of (2.34) to give

$$S_n \bar{C}_{n-1} + (T_n + \Omega)\bar{C}_b = V_n \tag{2.56}$$

as was the case for 2D conditions.

At the upper boundary the constant mass boundary condition in 3D is given by (2.6) which for the case of a rectangular landfill of length L and width W can be written

$$c_{LF}(t) = c_0 - \frac{1}{WLH_f}\int_0^t \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} f_T \, dx \, dy \, d\tau \tag{2.57}$$

and so the concentration on the nodal plane $z = z_0$ is given by

$$\left.\begin{aligned} c_T = c(x, y, z = z_0) &= c_{LF}(t) \qquad |x| < L/2, \ |y| < W/2 \\ c_T = c(x, y, z = z_0) &= 0 \qquad |x| > L/2, \ |y| > W/2 \end{aligned}\right\} \tag{2.58}$$

Taking the Laplace and Fourier transforms of (2.58) then gives

$$\bar{C}_T = \frac{1}{4\pi^2}\int_{-L/2}^{L/2}\int_{-W/2}^{W/2} \bar{c}_{LF} \, e^{-i\xi x - i\eta y} \, dx \, dy$$

$$= \bar{c}_{LF}\frac{LW}{4\pi^2}\frac{\sin(\xi L/2)}{(\xi L/2)}\frac{\sin(\eta W/2)}{(\eta W/2)} \tag{2.59}$$

where $\bar{c}_{LF}$ is as yet unknown.

The quantity $\bar{\chi} = \bar{F}_T$ can then be determined by solving (2.34) with the appropriate base boundary conditions and α, β, E, Ω defined by (2.54b), (2.54c) and (2.55b) for the case where

$$\bar{c}_T = \frac{LW}{4\pi^2}\frac{\sin(\xi L/2)}{(\xi L/2)}\frac{\sin(\eta W/2)}{\eta W/2}. \tag{2.60}$$

Taking the Laplace transform of (2.57) and invoking the Fourier inversion theorem the procedure described for the 2D case may be followed to give

$$\bar{c}_{LF} = \frac{WLH_f c_0}{sLWH_f + \Lambda} \tag{2.61a}$$

where

$$\Lambda = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{4\sin(\xi L/2)\sin(\eta W/2)}{\xi\eta}\bar{\chi} \, d\xi \, d\eta \tag{2.61b}$$

The reference concentrations $\bar{c}_{rj}$ at the nodal plane can now be obtained from the Fourier inversion theorem, viz

$$\bar{c}_{rj} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{C}_j \, e^{i\xi x + i\eta y} \, d\xi \, d\eta \qquad 1 \leqslant j \leqslant n \tag{2.62}$$

by numerical quadrature. The transform coefficients $\bar{C}_j$ in (2.62) are obtained by solving (2.34) subject to the appropriate boundary conditions (2.37) or (2.38) and the top boundary condition (2.60) by adopting the values of α, β, E, Ω defined by (54b), (54c) and (55b). At the same time the quantity Λ can also be determined from (2.61b). The concentrations $\bar{c}_{LF}$ can then be calculated from (2.61a) and hence the concentrations at the nodal plane are given by (2.51). The Laplace transform can then be numerically inverted using Talbot's algorithm.

2.7 RESULTS

The theory described in this chapter is amenable to the analysis of contaminant migration for a wide range of geometries and combinations of conditions. Rowe *et al.* [8] have shown how this approach can be used to determine the coefficient of hydrodynamic dispersion (which includes the effects of molecular diffusion) and the sorption potential (ρK) from the back analysis of results from a single laboratory column test. The application of the theory for the analysis of field problems has been demonstrated by Rowe *et al.* [8] and Quigley and Rowe [13].

The theory can also be readily used to perform sensitivity studies to indicate the effects of uncertainty regarding design parameters on the expected concentration of contaminant at proposed monitoring points beneath or adjacent to waste disposal sites. One problem of particular interest is that where the waste disposal site is separated from an underlying aquifer by a thin layer of relatively impermeable soil. This situation may arise naturally as a result of glaciation. For example, in Southern Ontario, Canada, it is not uncommon to encounter

layers of clayey till and clay which are separated by quite thin aquifers (i.e. less than 1 m thick). Thus, construction of a waste disposal site within the clayey till or clay layer may potentially cause contamination of the underlying aquifer. A similar situation arises when a compacted clay liner is constructed over an otherwise exposed aquifer. In either case, the effectiveness of the clayey 'liner' will be primarily assessed by considering the concentrations of contaminants (denoted as c_{base}) at various points within the aquifer.

To illustrate some of the features of the proposed theory, consider the hypothetical case of a landfill separated from a 1-m-thick aquifer by a 2-m-thick layer of clay. In cases such as this, it is often convenient to consider migration of contaminant within a plane section passing through the centre of the landfill and parallel to the direction of the superficial velocity v_b within the aquifer.

The simplest of the proposed analytical models which could be used for this case considers vertical 1D diffusion–dispersion advection within the clay layer but allows for horizontal contaminant transport in the aquifer by treating this as a boundary condition (see Section 2.4). Using this approach and adopting the parameters given in Figure 2.2, we can readily examine the effects of the height of leachate, the sorption potential (ρK) and the superficial velocity (v_a) upon the concentration of contaminant, c_{base}, within the aquifer directly beneath the landfill.

For example, if it is assumed that the concentration of contaminant within the landfill is a constant and independent of time (i.e. if the equivalent height of leachate H_f is assumed to be infinite), then the concentration in the aquifer increases with time until it reaches a steady-state value as shown in Figure 2.2. Comparing the results given for a conservative contaminant (i.e. one which does not react with the soil: $\rho K = 0$) and a reactive contaminant (with $\rho K = 10$), it is seen that for this case ($H_f = \infty$) sorption increases the time required to reach the final steady-state concentration but has no effect whatsoever on the magnitude of the final concentration. This finding is well recognized and would be predicted by most classical formulations of the problem. However, this case does not represent a very realistic situation.

The height of leachate, H_f, in most landfills lies in the range from 0.5–10 m, with value of 1 to 5 m being most common. Taking a value of H_f of 1 m, Figure 2.2 shows the variation in base concentration with time for a conservative ($\rho K = 0$) and non-conservative ($\rho K = 10$) contaminant. In both cases the base concentration increases with time until a peak concentration is reached and then decreases for subsequent time. The steady state is for zero concentration everywhere. Also it can be seen that when a realistic (finite) height of leachate is used in the analysis, sorption substantially reduces (in this case by almost a factor of 10) the magnitude of the peak concentration ever reached in the aquifer.

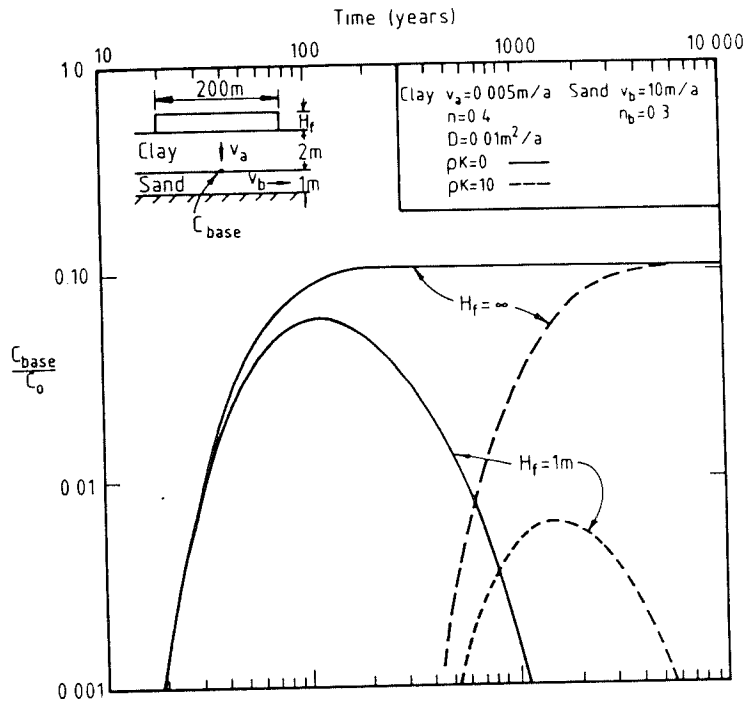


Figure 2.2 Variation in base concentration with time showing the effect of height of leachate (H_f) and sorption (ρK): 1D Analysis.

Intuitively, one would expect that the advective (superficial) velocity v_a in the clay would have a significant effect on the concentrations of contaminant within the aquifer. That this is so is illustrated by the results given in Figure 2.3. Recognizing the role of advection in the transportation of contaminant, it is clearly desirable to design landfills to minimize any downward seepage and in some cases it may be desirable to design a system which gives rise to upward flow from the aquifer into the landfill. However, it should be emphasized that contamination of the aquifer can occur due to downward diffusion even when there is opposing upward seepage into the landfill (e.g. see the results for $v_a = -0.005$ m/a in Figure 2.3).

Provided that the mass of contaminant is finite, there will be a maximum (peak) concentration within the aquifer (c_{bmax}) which will occur at some time t_{max}. The magnitude and time of occurrence of this maximum will be the primary quantity of interest in the design and evaluation of landfills separated from aquifers by either a natural or man-made clay liner. Among other things, these quantities c_{bmax} and t_{max} will depend on the height of leachate (H_f) and the advective velocity within the clay (v_a) as illustrated in Figure 2.4. A com-

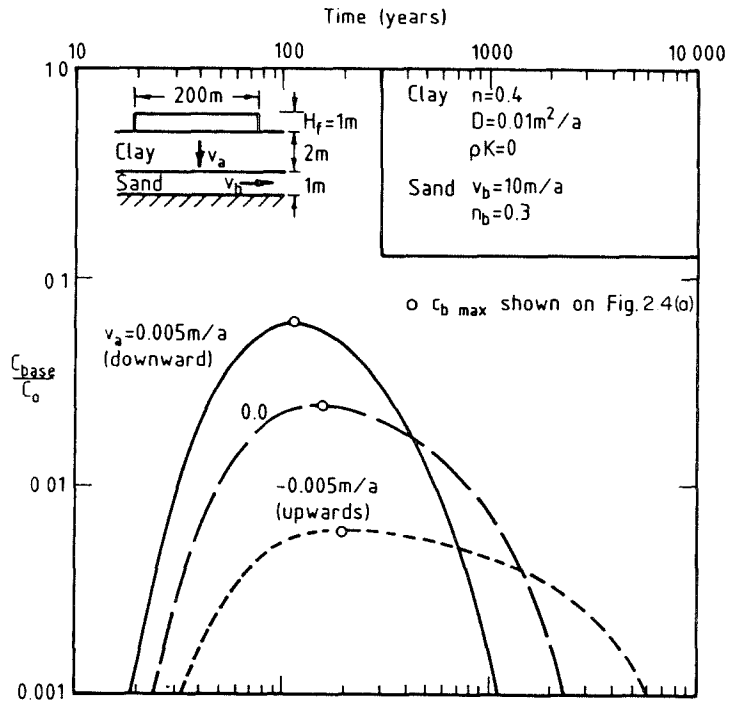


Figure 2.3 Variation in base concentration with time showing the effect of advective velocity v_a: 1D analysis.

parison of Figure 2.4(a) ($\rho K = 0$) and Figure 2.4(b) ($\rho K = 10$) clearly shows that the effect of the height of leachate is substantially greater for a reactive species ($\rho K = 0$), this appears to be true irrespective of the advective velocity, v_a.

The modified 1D approach (Section 2.4) used in the preceding example is the simplest of the proposed models which could be used. To make the problem tractable within this 1D framework, it was necessary to assume that the concentration within the aquifer directly beneath the landfill was spatially homogeneous at all times and that mass transport out of the aquifer directly beneath the landfill was only due to the horizontal advective velocity. This is equivalent to assuming that the coefficient of hydrodynamic dispersion in the aquifer is infinite directly beneath the landfill and zero at all points outside the boundary of the landfill. This is clearly an oversimplication of the actual situation and this raises the question 'what is the effect of the aquifer model upon predicted concentrations?'

If the 2D or 3D formulations are adopted (Sections 2.5 and 2.6), the conditions in the aquifer can be modelled in the following ways.

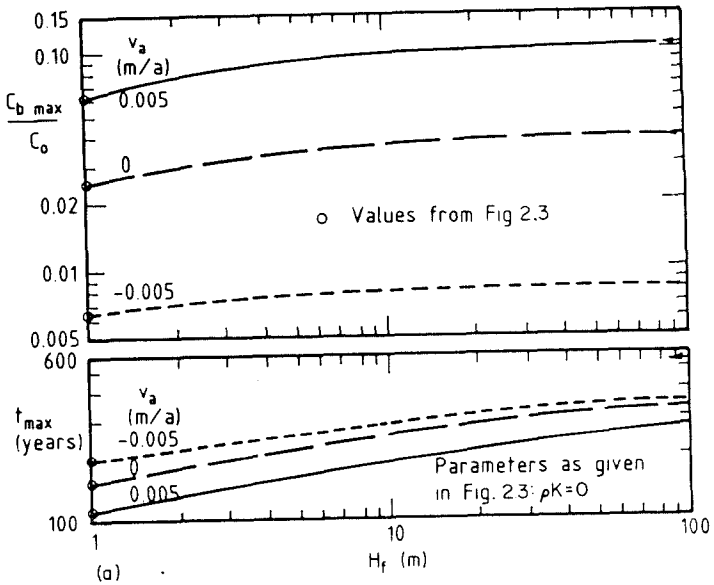


Figure 2.4 (a) Effect of leachate height H_f upon maximum base concentration: and time to reach maximum base concentration: 1D analysis: $\rho K = 0$.

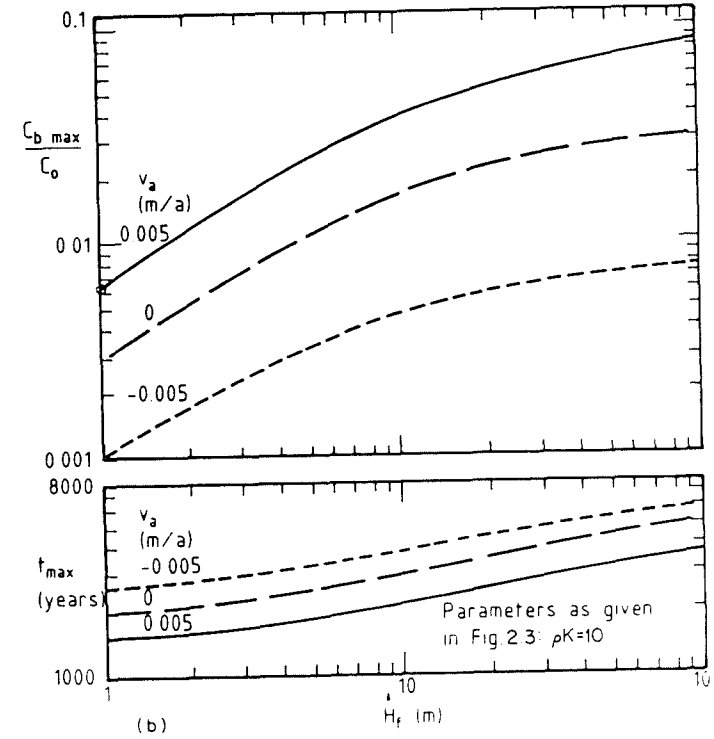


Figure 2.4(b) Effect of leachate height upon maximum base concentration and time to reach maximum base concentration: 1D analysis, $\rho K = 10$.

(1) As a boundary condition (see Section 2.3). This approach allows for spatial variations in concentration within the horizontal plane of the aquifer as well as advective dispersive transport within the aquifer itself. Thus this advective dispersive transport will depend on the horizontal velocity within the aquifer v_{bx}, v_{by} and the coefficient of hydrodynamic dispersion in the acquifer (D_{Hx}, D_{Hy}). However, this approach does assume that the concentration in the acquifer is uniform in the vertical direction (i.e. $D_v = \infty$) and that the acquifer is underlain by an inpenetrable boundary (i.e. zero flux across this boundary).

(2) As a physical layer having prescribed velocity components v_{bx}, v_{by} and coefficients of hydrodynamic dispersion D_v, D_{Hx}, D_{Hy}. This approach allows for spatial variations in concentration both vertically and horizontally within the aquifer. Treating the aquifer as a physical layer (i.e. in a manner similar to the clay but with different parameters) permits us to examine two cases:

(i) where the aquifer is underlain by an impenetrable boundary (as we assumed in (1) above); or
(ii) where the aquifer is underlain by an additional layer (or layers) of clay (and/or sand).

Considering firstly, case (i) where the aquifer is assumed to be underlain by an impenetrable boundary, 2D analyses were performed for the problem shown in Figure 2.5 using the parameters given in Table 2.1. Figure 2.5 shows

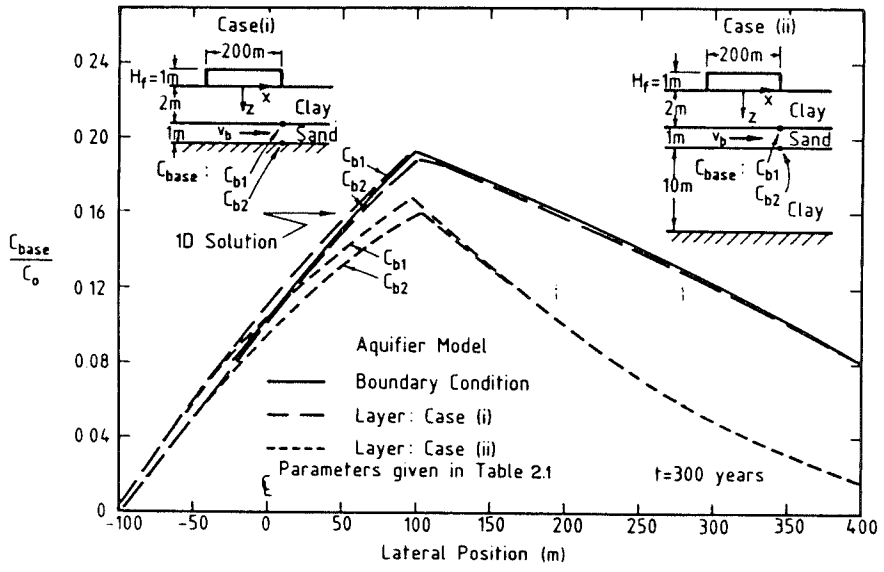


Figure 2.5 Concentration plume in aquifier at 300 years.

Table 2.1 Parameters used to obtain Figures 2.5 and 2.6

Layer	Quantity	Symbol (units)	Value
Clay	Vertical advective velocity	v_a(m/a)	0.0
	Porosity	n	0.4
	Sorption potential	ρK	0.0
	Coefficient of hydrodynamic dispersion (horizontal and vertical)	D(m^2/a)	0.01
Sand	Horizontal advective velocity	v_b(m/a)	1.0
	Porosity	n_b	0.3
	Sorption potential	ρK	0.0
	Coefficient of hydrodynamic Dispersion:		
	Horizontal	D_H(m^2/a)	1.0
	Vertical (layered case)	D_v(m^2/a)	0.2
	Vertical (boundary condition)	D_v(m^2/a)	∞

the concentration plumes obtained at $t = 300$ years using methods (1) and (2) above. Method 1 implicitly assumes that the concentration c_{base} is vertically uniform within the aquifer. Method 2 makes no *a priori* assumption regarding the spatial variation of concentration and the calculated values at the top (c_{b1}) and bottom (c_{b2}) of the aquifer are both shown in Figure 2.5. For the parameters considered, there is relatively little vertical variation in concentration within the aquifer and the results obtained by treating the aquifer as a physical layer (Method 2) closely bound the results from the computationally simpler approach where the aquifer is treated as a boundary condition (Method 1).

Now let us consider case (ii) where the aquifer is underlain by an additional layer of clay (in this case 10 m of clay is assumed to be resting on an impenetrable base). It is found that (see Figure 2.5) the concentration plume is almost the same as that obtained for case (i) near the upstream edge of the landfill but gives rise to considerably smaller concentrations near the downstream edge of the landfill and at points outside the landfill ($x \geqslant 100$ m). This decrease in concentration represents a natural attenuation as mass is transported into the lower clay layer by diffusion. The results given at the top and bottom of the aquifer indicate that there is a small concentration gradient in the vertical direction within the aquifer.

Analyses similar to those performed to obtain Figure 2.5 were repeated for different times to give the variation in concentration with time. Figure 2.6 summarizes the results for a point beneath the downstream edge of the landfill ($x = 100$ m) and a point well outside the landfill ($x = 400$ m). For the sake of

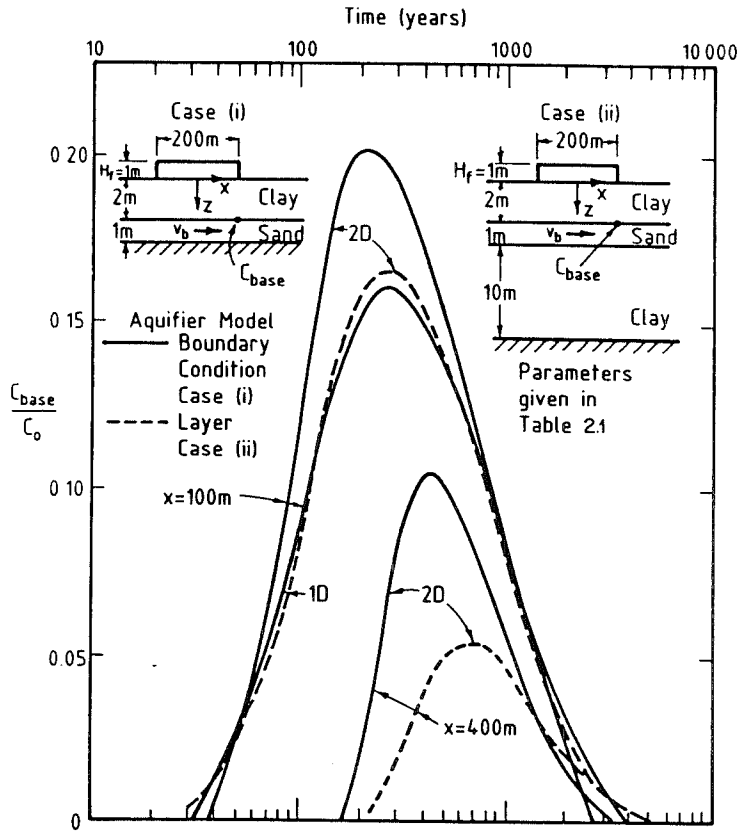


Figure 2.6 Concentration in aquifer beneath edge of landfill ($x = 100$ m): effect of model.

clarity, only the concentrations calculated at the top of the aquifer (c_{b1}) are shown. A number of observations can be made regarding these results.

Firstly, it is evident that there is a natural attenuation of contaminant plume as it advances along the aquifer ($x > 100$ m). The attenuation arises due to diffusion of contaminant into the adjacent clay. Thus the maximum concentration reached at $x = 400$ m is substantially less (by at least a factor of 2) than that obtained at the edge of the landfill $x = 100$ m.

Secondly, consideration of possible diffusion into a clay layer beneath the aquifer (case ii) gives rise to additional attenuation of contaminant concentration. At the edge of the landfill ($x = 100$ m), diffusion into the lower clay layer reduces the maximum concentration (compared to case i) by approximately 20%. The effect increases with distance away from the landfill and at $x = 400$ m diffusion into the lower clay reduces the maximum concentration by

almost a factor of two. Thus the modelling of the aquifer as a boundary condition in a 2D analysis provides a conservative estimate of the contaminant concentrations within the aquifer.

The modified 1D analysis which also treats the aquifer as a boundary condition gives an estimate of the concentration that may be expected in the aquifer directly beneath the landfill. The results obtained for the present problem are shown in Figures 2.5 and 2.6. Comparing these results with those obtained at $x = 100$ m from the 2D analysis indicates that for this case the modified 1D approach slightly overestimates the time required to attain the maximum concentration and underestimated the magnitude of the maximum concentration. However, in this case and a number of other cases examined [6] the discrepancy, which was typically less than 30%, may be acceptable in preliminary calculations.

It can be shown analytically that the total mass present in the aquifer at any time t, for any base velocity v_b, is equal to the mass into the aquifer calculated from the 1D solution for $v_b = 0$, i.e.

$$m(t) = \int_{-\infty}^{\infty} \int_0^t f_z(x, z_n, \tau)\, d\tau\, dx = L \int_0^t f_{1D}(z_n, \tau)\, d\tau$$

in which L is the 'width' of the landfill;

$f_{1D}(z_n, \tau)$ is the flux into the aquifer at time τ determined from the 1D solution for $v_b = 0$;

$f_z(x, z_n, \tau)$ is the flux into (or out of) the aquifer at time τ and position x from the 2D solution for the actual value of v_b.

It should be noted that the total mass present in the aquifer ($-\infty \leqslant x \leqslant \infty$) at any time t will be less than the total mass that has passed into the aquifer directly beneath the landfill ($-L/2 < x < L/2$) up to this time t. The difference between these masses is due to diffusion from the aquifer back into the clay outside the landfill (i.e. $|x| > L/2$). At large time the mass of contaminant which has diffused into the clay is quite large. It is also interesting to observe that the total mass remaining in the aquifer at any given time is independent of the base velocity v_b (although the total mass which has, at some time, moved into the base does depend on v_b).

2.8 OTHER FINDINGS

The problem of a clayey layer underlain by a thin aquifer examined in the previous sections has been considered in more detail by Rowe and Booker [2–6]. In these studies consideration was given to the effects of the height of leachate H_f, the advective velocity v_a, the horizontal advective velocity (referred to as the 'base velocity', v_b), the coefficient of hydrodynamic dispersion in the vertical and horizontal direction (D_H, D_v), the sorption potential (ρK),

the thickness of the clay layer, the thickness of the aquifer, and the dimensions of the landfill. As a result of these studies, it was observed that:

(1) If the base velocity is greater than zero, then the concentration of pollutant in the aquifer beneath the liner will reach a maximum value, $c_{b\max}$, at some time $t_{\max}$, and will decrease for greater times. This maximum value can be used in design to ensure that the contamination of the groundwater never exceeds a specified level.

(2) Geochemical reactions (sorption) can greatly affect the magnitude of the maximum base concentration as well as the time required to reach this maximum. The sorption potential will depend on the species of contaminant being considered and the chemical properties of the clay. For reactive species, the clay absorbs pollutant when the concentration is increasing. Once the concentration at a point reaches a maximum value, the sorbed contaminant may remain fixed to the clay or, in the worst case, may be released back into the pore fluid as the concentration drops. This beneficial buffering role for some species of pollutant is only apparent if the finite mass of pollutant in the landfill is considered.

(3) The effect of sorption is greatest for low to moderate volumes of leachate (i.e. leachate height less than 10 m). Sorption may be particularly important in designing for hazardous substances such as NH_4 and heavy metals although careful consideration of the chemical properties of the clay is necessary.

(4) The maximum base concentration can be decreased by increasing clay liner thickness. However, the design of the liner may be optimized by also considering the effects of leachate height, base velocity and any geochemical reaction.

(5) The consideration of advection velocity does not alter the trends described above. Downward advective transport increases the maximum base concentration and decreases the time required to reach this concentration.

(6) Diffusion of contaminant from the aquifer into the surrounding clay will provide natural attenuation of contaminant in the aquifer. The maximum concentration reached at any point outside the boundaries of the landfill will never reach the maximum value at the edge of the landfill. This phenomenon may be used in specifying the buffer zone required around a landfill to ensure the groundwater quality outside the buffer is never degraded below allowable levels due to contamination from the landfill.

(7) The magnitude of the maximum concentration in the aquifer beneath the landfill decreases:

(a) with decreasing mass of contaminant in the landfill; and
(b) with increasing base velocity.

(8) There is a critical base velocity which will result in the maximum concentration of contaminant at a point in the aquifer away from the landfill. Velocities either larger or smaller than the critical velocity will result in a smaller peak concentration in the aquifer at the point of interest. Thus it is not necessarily conservative to design for only the maximum and minimum expected velocities in the aquifer and a range of values between these limits should be considered.

(9) Except for the case where the dispersion coefficient is very high or the advective velocity is very low, advection is the predominant mechanism for mass transport within the aquifer.

(10) Increasing thickness of the aquifer (all other things being equal) tends to decrease the concentration at the edge of the landfill. This is primarily a result of increased dilution due to a correspondingly higher flow of water through the aquifer. However, at points well away from the landfill, the calculated maximum concentration may actually increase with increasing thickness of the aquifer (again, all other things being equal) because the relative diffusion into the surrounding clay layers is reduced.

(11) Increasing the dimensions of the landfill increases the maximum concentration at both the edge and at points 'downstream' (with respect to the direction of advective flow within the aquifer) of the landfill. This is a result of an increased mass loading of the aquifer which arises from a larger total mass of contaminant within the landfill.

2.9 CHOICE OF ANALYSIS

The semianalytic methods of analysis developed in this chapter assume that the soil stratigraphy consists of horizontal layers in which there is no variation in properties in the horizontal plane. Clearly, this will not be the case in all field situations. Nevertheless, in many practical cases the stratigraphy is sufficiently regular that it can be reasonably idealised as having horizontal layering. For problems involving complicated stratigraphy, it will usually be necessary to adopt a more general numerical technique (e.g. the finite element method) but even in these cases the techniques proposed here may be useful in developing preliminary estimates of concentrations as well as for providing benchmark results against which finite element calculations can be checked.

The primary advantage of the semianalytic techniques as compared to alternative numerical approaches, is that it is possible to directly determine the concentration of contaminant at specific points and times without determining the entire concentration field at previous times. In many applications, it is really only necessary to determine the magnitude and time of occurrence of the maximum expected base concentration at a few specific points which will usually be specified as monitoring points in the aquifer (e.g. beneath the landfill and at a few points outside the landfill). Using a 'binary chop' algorithm (which

can be readily incorporated into the computer program), the maximum concentration expected to occur at the point can usually be determined to a precision of 0.1% or better by evaluating the concentration at these points for seven times. Clearly, even fewer times would be required if one has a reasonable initial estimate of the time at which the maximum will occur. Since the data preparation time is also quite small, sensitivity studies can be easily performed to determine the effect of uncertainty regarding input parameters (e.g. v_b, v_a, ρK, etc) upon the calculated maximum concentrations at the specified monitoring points.

The question of whether the 1D, 2D or 3D formulation is adopted will, of course, depend on the specific problem. The computational effort involved in performing a 3D analysis may be less than that involved in performing a 3D finite element or finite difference analyses but it is, nevertheless, sufficiently large to predicate against performing 3D sensitively analyses for most practical applications.

Provided reasonable engineering judgement is exercised in selecting appropriate cross sections for analysis, it is considered that a 2D analysis will be adequate for the vast majority of applications. A full 3D analysis may be warranted to check the most critical case identified from the 2D analyses.

The modified 1D approach is ideally suited to the analysis of laboratory column tests and other similar 1D situations. It may also be useful for:

(1) Obtaining an estimate of the contaminant migration through the clay liner at the sides of the landfill.

(2) Identifying when significant concentrations are to be expected in the aquifer.

(3) Providing an initial estimate of the magnitude of the peak concentration (and its time of occurrence) in the aquifer beneath the landfill, thereby minimizing the amount of computation required to get accurate values for these quantities from a subsequent 2D analysis.

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NOTATION

A	plan area of the landfill
$c = c(x, y, z, t)$	concentration of contaminant at any point (x, y, z) within the clay layer at time t
$c_T = c(x, y, z_0, t)$	concentration of contaminant at the top of the clay layer ($z = z_0$)
$c_b = c_{base}$	$= c(x, y, z_n, t)$ concentration of contaminant within the base aquifer at position (x, y)
c_{bmax}	maximum concentration of contaminant ever reached at position (x, y) in the base aquifer
c_I	initial (background) concentration
c_{LF}	concentration of contaminant within the landfill
c_0	initial concentration of contaminant within the landfill
$\bar{c}$	Laplace transform of c
C	Fourier transform of c
D_{xx}, D_{yy}, D_{zz}	coefficient of hydrodynamic dispersion (including diffusion and mechanical dispersion) in the x, y and z Cartesian directions
D	isotropic coefficient of hydrodynamic dispersion

D_{Hx}, D_{Hy}	horizontal coefficient of hydrodynamic dispersion in x, y directions in the base aquifer
$f_x = f_x(x, y, z, t)$	the flux in the x direction
$f_y = f_y(x, y, z, t)$	the flux in the y direction
$f_z = f_z(x, y, z, t)$	the flux in the z direction
$f_T = f_z(x, y, z_0, t)$	the vertical flux entering the top of the clay layer at position (x, y)
$f_b = f_z(x, y, z_n, t)$	the vertical flux entering the base aquifer at position (x, y)
$\bar{f}_x, \bar{f}_y, \bar{f}_z, \bar{f}_T, \bar{f}_b$	Laplace transforms of f_x, f_y, f_z, f_T, f_b
F_x, F_y, F_z, F_T, F_b	Fourier transforms of f_x, f_y, f_z, f_T, f_b
H_f	equivalent height of leachate (volume of leachate per unit plan area)
h	thickness of aquifer
K	distribution coefficient
L	width of the landfill parallel to the velocity v_b
n	porosity of the clay layer
n_b	porosity of the base aquifer
t, τ	time
v_x, v_y, v_z	horizontal (x and y) and vertical (z) components of the seepage velocity
v_a	apparent (Darcy, superficial) vertical = nv_z
v_b	apparent (Darcy, superficial) horizontal velocity in the base aquifer
v_{bx}, v_{by}	components of the apparent velocity in the base aquifer
x, y	horizontal distance from the centre of the landfill
z	vertical distance below the base of the landfill
ρ	dry density of the soil soilds
λ	thickness of a layer

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A finite layer technique for calculating three-dimensional pollutant migration in soil

R. K. ROWE* and J. R. BOOKER†

A technique for the analysis of two- and three-dimensional pollutant migration through a layered soil medium is described. An earlier solution for plane diffusion in a single homogeneous layer of soil is extended using the finite layer method for general three-dimensional diffusion. Particular attention is focused on the effects of horizontal advective velocity and coefficient of hydrodynamic dispersion within the aquifer together with the thickness of the aquifer. A parametric study is presented to demonstrate some characteristics of contaminant migration in a layered soil system, taking into account the fact that the surface concentration does not remain constant because of contaminant transport into the deposit. The advantages of the approach are most pronounced when attempting to determine concentrations away from the landfill at modest to large times.

Dans cet article une technique d'analyse bidimensionnelle et tridimensionnelle de la migration des pollutions à travers un sol multicouches est décrit. Une méthode pour la diffusion plane dans une seule couche homogène de sol est développée à l'aide de la méthode des couches finies pour la diffusion tridimensionnelle générale. L'attention est portée plus particulièrement sur les effets de la vitesse d'advection horizontale et au coefficient de dispersion hydrodynamique à l'intérieur de l'aquifère et à son épaisseur. Une étude des paramètres est présentée pour démontrer quelques caractéristiques de la migration des pollutions dans un sol multicouches, en égard au fait que la concentration superficielle ne reste pas constante, en raison du transfert des pollutions dans le dépôt. L'avantage principal de la méthode consiste en ce qu'elle aide à la détermination des concentrations à long ou à court terme.

KEYWORDS: analysis; clays; computation; groundwater; industrial waste; water flow.

NOTATION

A_{LF} average plan area of the landfill
c concentration of contaminant
c_A, c_0 initial concentration of contaminant within the landfill
c_b concentration of contaminant within an aquifer
$c_{b\,max}$ maximum concentration ever reached at a particular point in the aquifer
c_{LF} concentration of contaminant within the landfill
D isotropic coefficient of hydrodynamic dispersion
D_{Hi} coefficient of hydrodynamic dispersion in the horizontal plane for aquifer i
D_{vi} coefficient of hydrodynamic dispersion in the vertical plane for aquifer i
D_{xx}, D_{yy}, D_{zz} coefficient of hydrodynamic dispersion (including diffusion and mechanical dispersion) in the three Cartesian directions
f mass flux
f_b mass flux into base aquifer
f_{LF} mass flux from the landfill into the soil
h_i thickness of aquifer i
H_f height of leachate (volume of leachate per unit plan area)
H_i thickness of clay layer i
K distribution coefficient
L width of the landfill parallel to the velocity v_b
n porosity
n_b porosity of the base aquifer
v seepage velocity
v_a apparent (Darcy, superficial) velocity
v_{bi} apparent (Darcy, superficial) velocity in aquifer i
z distance below the base of the landfill
ρ bulk density of the soil solids

INTRODUCTION

The design and subsequent environmental approval of landfill sites involves ensuring that the groundwater in regions outside the boundaries of the landfill (and any associated buffer zones) will not be contaminated. In effect, this

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means limiting the concentration of contaminant species in the groundwater to acceptable levels. Regulatory authorities often require the use of a liner which is designed to minimize contamination of the surrounding groundwater. Frequently, these liners take the form of a natural deposit of clay or a compacted clay liner.

The movement of pollutants through relatively impermeable clayey soils is quite slow; however, it is conceivable that significant pollution might occur in the long term and so it is important that such disposal sites should be designed to prevent the possible contamination of the groundwater system in both the short and the long term.

The main factors which govern contaminant migration are advection, dispersion, diffusion and chemical reaction. For many applications, the advective transport is small compared with that due to dispersion (e.g. Crooks & Quigley, 1984). This situation has been investigated using both analytic and numerical techniques (see Freeze & Cherry (1979) for the general background and Anderson (1979) for a detailed review). However, existing solutions have not considered a number of important practical situations.

More recently, Rowe & Booker (1985) have developed a one-dimensional analysis which takes account of the fact that the concentration of leachate in the landfill will decrease as leachate is transported into the clay and which allows for the possible presence of a more permeable underlying stratum (aquifer) beneath the clay deposit. This analysis has been extended by Rowe & Booker (1984a) to include the effects of several different soil layers.

The one-dimensional analysis may be adequate provided that the width of the landfill is large compared with the depth of the clay layer and transport is predominantly vertical. If, however, the plan dimensions of the landfill are comparable with the depth of the layer both horizontal and vertical transport may occur.

In a recent paper, Rowe & Booker (1984b) have developed a solution for plane diffusion in a single homogeneous layer of soil. In this Paper, this approach is extended, using the finite layer method (Cheung, 1976; Rowe & Booker, 1982; Booker & Small, 1982a, 1982b) to account for general three-dimensional diffusion in a horizontally layered soil deposit. A limited parametric study will then be presented to demonstrate some of the important characteristics of contaminant migration in a layered soil system.

THEORY: GOVERNING EQUATIONS

The transport of substances through a saturated clay can often be approximated by a Fickian-type law (e.g. Gillham & Cherry, 1982), having the form

$$f = ncv - nM_D \nabla c \tag{1}$$

$$M_D = \text{diag}\,(D_{xx}, D_{yy}, D_{zz})$$

where f is the flux and c the concentration of the dispersing substance, n is the effective porosity of the clay, M_D is the matrix of 'coefficients of hydrodynamic dispersion' (incorporating the effects of molecular diffusion and mechanical dispersion) and v is the seepage velocity. The velocity $v_a = nv$ will be used to describe advection when discussing the results. The quantity v_a is variously called the advective velocity, the superficial velocity, the apparent velocity or the discharge velocity. For simplicity of presentation, the term advective velocity will be used throughout this Paper.

Consideration of mass balance shows that

$$\nabla^T f + n \frac{\partial c}{\partial t} + g = 0$$

where the quantity g takes account of the possibility of some of the contaminant being adsorbed on to the clay skeleton. For equilibrium-controlled ion exchange where the concentration of the exchange ion is relatively low, the adsorption of this species may be approximated by a linear relationship of the form

$$g = \rho K \frac{\partial c}{\partial t} \tag{2}$$

where ρ is the bulk density of solid and K is the distribution coefficient. The distribution coefficient K may often be estimated from the results of a laboratory column test (Rowe, Caers, Booker & Crooks, 1984) or may be determined independently (Griffin, Cartwright, Shimp, Steele, Ruch, White, Hughes & Gilkeson, 1976). It should be determined over a representative range of concentrations which reflect the likely field variation.

For an isotropic homogeneous layer in which the pore fluid velocity is uniform, equations (1) and (2) can be combined to give

$$\nabla^T(M_D \nabla c) - v^T \nabla c = \left(1 + \frac{\rho K}{n}\right) \frac{\partial c}{\partial t} \tag{3}$$

SOLUTION USING INTEGRAL TRANSFORMS

The transport process is governed by equations (1)–(3) together with the boundary conditions and the condition that any initial background concentration is in equilibrium. For simplicity of presentation, attention will be restricted to the case in which the seepage velocity throughout a particu-

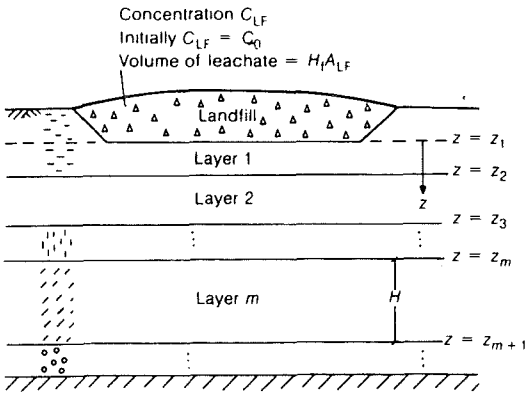

Fig. 1. Cross-section through a landfill with average plan area A_{LF}

lar layer is uniform and in the vertical direction so that

$$v = \left(0, 0, \frac{v_a}{n}\right)^T$$

(Extension of the solution for flow in the x and y directions is straightforward.)

Suppose now that the soil deposit is horizontally layered as shown in Fig. 1. In a typical layer m, $z_m \leqslant \bar{z} \leqslant z_p$ $(p = m + 1)$, equations (1)–(3) can be simplified by introducing a Laplace transform

$$(\bar{c}, \bar{f}) = \int_0^\infty (c, f) \exp(-st)\, dt \tag{4}$$

and a repeated Fourier transform

$$(C, F) = \frac{1}{4\pi^2} \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} (c, f) \times \exp[-i(\xi x + \eta y)]\, dx\, dy \tag{5}$$

The Fourier integral theorem shows that equation (5) is equivalent to the representation

$$(c, f) = \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} (C, F) \times \exp[i(\xi x + \eta y)]\, d\xi\, d\eta \tag{6}$$

When the transforms defined by equations (4) and (5) are applied to equations (1) and (3)

$$F_x = -i\xi n D_{xx} C \tag{7a}$$

$$F_y = -i\eta n D_{yy} C \tag{7b}$$

$$F_z = v_a C - n D_{zz} \frac{\partial C}{\partial z} \tag{7c}$$

$$D_{zz} \frac{\partial^2 \bar{C}}{\partial z^2} - (\xi^2 D_{xx} + \eta^2 D_{yy})\bar{C} - \frac{v_a}{n} \frac{\partial \bar{C}}{\partial z} = \left(1 + \frac{\rho K}{n}\right) s\bar{C} \tag{8}$$

where the properties n, D_{xx}, D_{yy}, D_{zz}, ρ and K are those of the mth layer.

For a local set of co-ordinates with origin at the top of layer m, it is a simple matter to show that equation (8) has the solution

$$\bar{C} = \bar{C}_m \frac{\exp(\alpha Z + \beta) - \exp(\beta Z + \alpha)}{\exp\beta - \exp\alpha} + \bar{C}_p \frac{\exp(\beta Z) - \exp(\alpha Z)}{\exp\beta - \exp\alpha} \tag{9}$$

where $\bar{C}_p$ and $\bar{C}_m$ denote the transformed concentrations on the node planes $z = z_p$ and $z = z_m$ respectively, $H = z_p - z_m$ is the depth of the layer, $Z = z/H$, and $\lambda = \alpha$ and $\lambda = \beta$ are the roots of the equation

$$\lambda^2 - 2P\lambda + Q = 0 \tag{10}$$

with

$$P = \frac{v_a H}{2nD_{zz}}$$

$$Q = \frac{\xi^2 H^2 D_{xx} + \eta^2 H^2 D_{yy}}{D_{zz}} + \left(1 + \frac{\rho K}{n}\right) \frac{sH^2}{D_{zz}}$$

so that

$$\alpha = P + (P^2 - Q)^{1/2}$$

$$\beta = P - (P^2 - Q)^{1/2}$$

where again the properties H, n, D_{xx}, D_{yy}, D_{zz}, ρ and K refer to those of the mth layer.

LAYER MATRIX

The transformed components of vertical flux $\bar{F}_p$ and $\bar{F}_m$ for the node planes $z = z_p$ and $z = z_m$ respectively may be calculated using equations (7)–(9) and it is found that

$$\begin{bmatrix} \bar{F}_m \\ -\bar{F}_p \end{bmatrix} = \begin{bmatrix} S_{11}^{(m)} & S_{12}^{(m)} \\ S_{21}^{(m)} & S_{22}^{(m)} \end{bmatrix} \begin{bmatrix} \bar{C}_p \\ \bar{C}_m \end{bmatrix} \tag{11}$$

where

$$S_{11}^{(m)} = \frac{nD_{zz}}{H} \frac{\beta \exp\beta - \alpha \exp\alpha}{\exp\beta - \exp\alpha}$$

$$S_{12}^{(m)} = \frac{nD_{zz}}{H} \frac{\alpha - \beta}{\exp\beta - \exp\alpha}$$

$$S_{21}^{(m)} = \frac{nD_{zz}}{H} \frac{(\alpha - \beta)\exp(\alpha + \beta)}{\exp\beta - \exp\alpha}$$

$$S_{22}^{(m)} = \frac{nD_{zz}}{H} \frac{\alpha \exp\beta - \beta \exp\alpha}{\exp\beta - \exp\alpha}$$

The matrix S in equation (11) is called the layer matrix for layer m.

Assembly of layer matrices

The layer matrix (11) can be used to solve the general problem for a layered system. To do this, at the interface of adjacent layers, the concentration will be continuous and the net vertical flux entering that particular node plane will be zero. Thus

$$S\bar{C} = F \tag{12}$$

where $C = (C_0, C_1, \ldots, C_N)^T$ is the vector of transformed nodal concentrations, $F = (F_0, 0, \ldots, -F_n)^T$ is the vector of transformed boundary fluxes and

$$S = \begin{bmatrix} S_{11}^{(0)} & S_{12}^{(0)} & 0 & \cdots \\ S_{21}^{(0)} & S_{22}^{(0)} + S_{11}^{(1)} & S_{12}^{(1)} & \cdots \\ 0 & S_{21}^{(1)} & S_{22}^{(1)} + S_{12}^{(1)} & \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

Equation (12) has to be modified to incorporate the boundary conditions.

Often, it is found that a given layered deposit will be underlain by a thin more permeable stratum of thickness h in which the groundwater flows with a uniform velocity v_b in the x direction. If it is assumed that the concentration in this stratum is virtually uniform across its width, it then follows from a consideration of conservation of mass that

$$c_b(x, y, t) = \int_0^t \left(\frac{f_b}{n_b h} - \frac{v_b}{n_b} \frac{\partial c_b}{\partial x} + D_{xxb} \frac{\partial^2 c_b}{\partial x^2} + D_{yyb} \frac{\partial^2 c_b}{\partial y^2} \right) dt \tag{13}$$

where $c_b(x, y, t)$ denotes the concentration in the underlying stratum, $f_b = f_N$ denotes the flux entering the base stratum from the clay layer and the subscript b is used to indicate properties of the base stratum.

If equation (13) is transformed using equations (4) and (5), it is found that

$$\bar{F}_N = \chi \bar{C}_N$$

where

$$\chi = h(n_b s + i\xi v_b + n_b D_{xxb} \xi^2 + n_b D_{yyb} \eta^2) \tag{14}$$

Suppose now, for simplicity, that the surface concentration c_0 is specified; then equations (12) and (14) become

$$S^* \bar{C}^* = F^* \tag{15}$$

where $\bar{C}^* = (\bar{C}_1, \bar{C}_2, \ldots, \bar{C}_N)^T$ is the vector of unknown transformed concentrations, $\bar{F}^* = (-S_{21}^{(0)} \bar{C}_0, 0, \ldots, 0)^T$ and

$$S^* = \begin{bmatrix} S_{22}^{(0)} + S_{11}^{(1)} & S_{12}^{(1)} & 0 & \cdots & \\ S_{21}^{(1)} & S_{22}^{(1)} + S_{11}^{(2)} & S_{12}^{(2)} & & \\ \vdots & \vdots & & & \\ 0 & 0 & & & \\ 0 & 0 & & \cdots & \\ & & \cdots & & \\ & & & S_{22}^{(N-1)} + S_{11}^{(N)} & S_{12}^{(N)} \\ & & & S_{21}^{(N)} & S_{22}^{(N)} + \chi \end{bmatrix}$$

Equation (15) can be used to determine $\bar{C}_1, \ldots, \bar{C}_N$ and hence, using equation (9), the complete distribution of $\bar{C}$. This represents the solution in transform space. The Fourier transforms can be inverted by numerical quadrature of equation (6) while the Laplace transforms can be inverted by an efficient algorithm developed by Talbot (1979). This then completes the solution.

ANALYSIS WHEN SURFACE CONCENTRATION VARIES WITH TIME

In many practical situations the surface concentration of the landfill does not remain constant but will vary because contaminant is being transported into the layered soil deposit.

To analyse this situation, consider a landfill occupying a region of plan area A_{LF} to an equivalent height H_f (where H_f represents the volume of leachate divided by the average plan area of the landfill). For simplicity, it is assumed that the concentration of leachate in the landfill does not vary throughout the landfill but may vary with time.

Now suppose that the total mass of landfill that has been placed up to time t is $M(t)$; then from a consideration of conservation of mass

$$A_{LF} H_f c_{LF}(t) = M(t) - \int_0^t \left(\int_{A_{LF}} f_{LF}(x, y, \tau) \, dx \, dy \right) d\tau \tag{16}$$

where A_{LF} is the area of landfill, $f_{LF}(x, y, \tau)$ is the flux leaving the landfill at the point (x, y) at time $t = \tau$.

Taking the Laplace transform of equation (16), it is found that

$$\bar{c}_{LF} = \bar{c}_A - \frac{1}{sA_{LF} H_f} \int_{A_{LF}} \bar{f}_{LF}(x, y) \, dx \, dy \tag{17}$$

If equation (17) is solved in terms of the yet unknown surface concentration c_{LF} and this solu-

tion is used to calculate the transformed surface flux

$$\bar{F}_{LF} = \Psi \bar{C}_{LF} \tag{18}$$

where Ψ is a known function of ξ and η and

$$\bar{C}_{LF} = \bar{c}_{LF}\, T$$

with

$$T = \frac{1}{4\pi^2} \iint_{A_{LF}} \exp\,[-i(\xi x + \eta y)]\, dx\, dy$$

Now

$$f_{LF} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\,[i(\xi x + \eta y)] F_{LF}\, d\xi\, d\eta$$

and so substituting into equation (18)

$$\bar{c}_{LF} = \frac{\bar{c}_A}{1 + \Lambda} \tag{19}$$

where

$$\Lambda = \frac{4\pi^2}{s A_{LF} H_f} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T^* T \Psi\, d\xi\, d\eta$$

and the asterisk denotes the complex conjugate.

Equation (19) enables the surface concentration c_{LF} to be determined and so the problem may be solved as outlined in the previous section.

RESULTS

The theory described in the previous section is applicable for a wide range of practical problems involving three-dimensional advective–dispersive transport through soil. The major computational effort involved is associated with the numerical inversion of the Laplace and Fourier transforms. Some care needs to be exercised to perform these inversions efficiently and accurately; however, the approach has significant advantages over alternative finite element and finite difference approaches in that it involves minimal data preparation and much less computational effort to obtain results of equivalent accuracy. The advantages of the proposed approach are most pronounced when it is attempted to determine concentrations away from the landfill at modest to large times (although the proposed procedure is computationally superior and easier to use for all the cases examined by the Authors).

The computational effort in this analysis, as with any other three-dimensional analysis, is quite large and makes the performance of a parametric study prohibitively expensive. However, the general effect of the parameters which would influence a full three-dimensional analysis can be illustrated by considering a typical two-dimensional cross-section such as that illustrated in Fig. 2. This figure shows a stratigraphy similar to that commonly encountered in areas subjected to recent glaciation. Here the landfill is separated from a relatively thin underlying aquifer by a layer of clay or clayey till. The aquifer is underlain by a second thicker layer of clay or clayey till which in turn is underlain by a second sand layer.

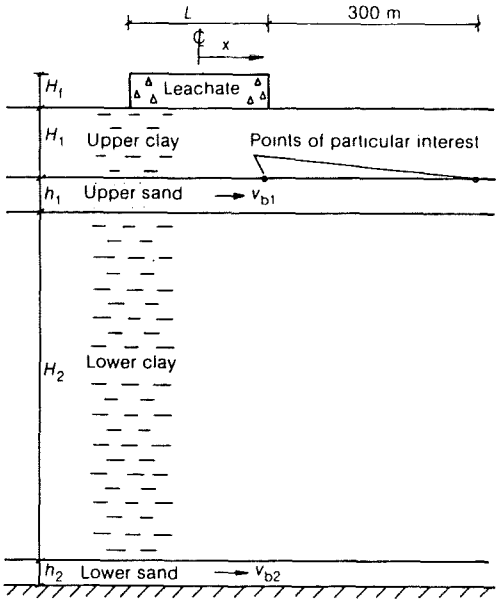

Fig. 2. Problem analysed (the parameters considered are listed in Table 1)

In such a situation, the designer is concerned with the concentration of contaminant in the aquifers (particularly the upper aquifer) at the edge of the landfill and at the boundary of the property. Specifically it is assumed (unless otherwise noted) that the deposit has parameters given in Table 1 for a landfill of length 200 m where it is further assumed that the boundary of the property is 300 m from the edge of the landfill (i.e. 400 m from the centre of the landfill).

In the analysis of problems similar to that shown in Fig. 2 (either in two or three dimensions) the major uncertainty is usually associated with the properties of the aquifer. In particular, the advective velocity v_b and the horizontal and vertical dispersion coefficients in the aquifer (D_H and D_v) may be difficult to determine and the effect of a reasonable range of these parameters should be considered.

Figure 3 shows the effect of the horizontal coefficient of hydrodynamic dispersion D_{H1} on the contaminant plume within the upper aquifer at a time 300 years after construction of the landfill (given $D_v = 0{\cdot}2$ m^2/year, $v_b = 1$ m/year). For a

Table 1. Parameters considered in the study

Layer	Symbol	Typical value*	Range considered
Landfill	L (m)	200	200–1000
	H_f (m)	1	1–10
Upper clay	D (m^2/year)	0.01	—
	n	0·4	—
	v_a (m/year)	0·0	—
	H_1 (m)	2·0	—
Upper sand	D_{H1} (m^2/year)	10·0	0·2–200
	D_{v1} (m^2/year)	0·2	—
	n	0·3	—
	v_{b1} (m/year)	1·0	0·1–10
	h_1 (m)	1·0	0·3–1·5
Lower clay	D (m^2/year)	0·01	—
	n	0·4	—
	v_a (m/year)	0·0	—
	H_2 (m)	10·0	—
Lower sand	D_{H2} (m^2/year)	10·0	—
	D_{v2} (m^2/year)	0·2	—
	n	0·3	—
	v_{b2} (m/year)	1·0	—
	h_2 (m)	0·3	—

* Value adopted unless otherwise noted.

low value of D_H (D_H = 0·2 m^2/year) there is essentially no contamination upstream of the landfill. The maximum concentration in the aquifer is at the edge of the landfill (x = 100 m) and the concentration decreases with increasing horizontal distance away from the landfill.

By comparison, a very high dispersion coefficient (D_{H1} = 200 m^2/year) gives rise to contamination of the aquifer upstream of the landfill. In this case the maximum concentration is not necessarily at the edge (for t = 300 years the maximum occurs approximately 10 m inside the landfill) and is somewhat less than that obtained for a lower value of D_{H1}. It should be noted, however, that the high value of the dispersion coefficient D_{H1} does give rise to a higher concentration at the property line (i.e. x = 400 m) at this time because it allows more rapid movement of contaminant away from the landfill.

Figure 3 shows the concentration plume at one point in time. Clearly the concentration at any point will vary with time as indicated in Fig. 4 for a point in the upper aquifer beneath the edge of the landfill (x = 100 m) and at the property line (x = 400 m). At both points, the concentration increases with time until at some time t_{max} a maximum concentration is reached at that point. For subsequent times, the concentration reduces until, at very large times, the concentration becomes negligible. The magnitude and time of occurrence of this peak concentration will depend on the geometry of the problem and the soil and leachate properties. In particular, however, it depends on the height of leachate H_f.

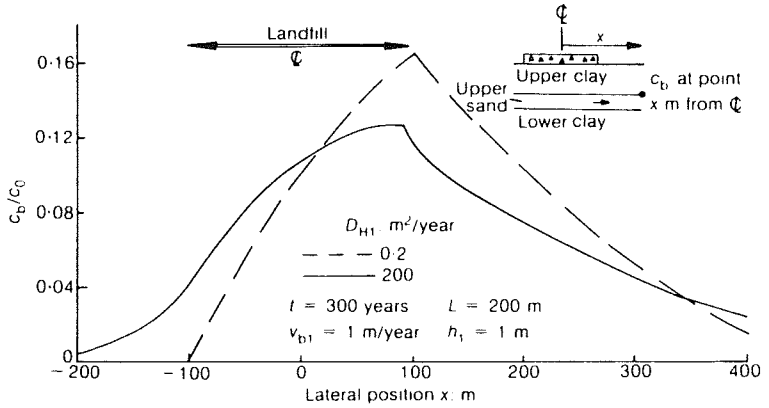

Fig. 3. Concentration plumes within the upper sand layer

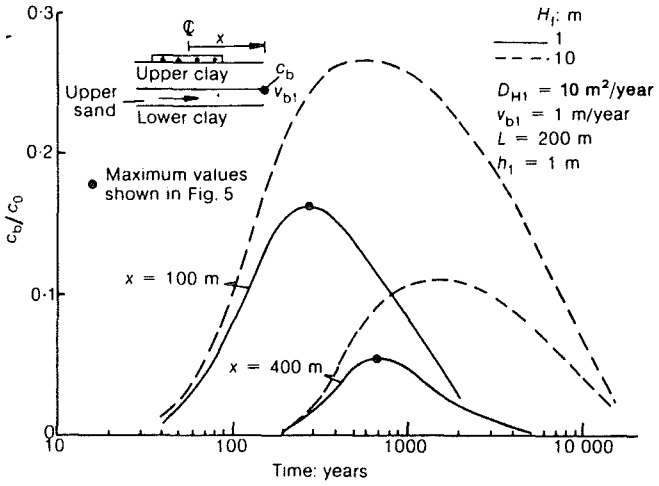

Fig. 4. Variation in concentration c_b in the upper sand layer with time for two locations (x = 100 m and x = 400 m)

The mass of contaminant contained with the landfill generally reaches a maximum value at the completion of the landfill. It may take a long time for this mass to be leached out of the solid waste; however, at some relatively short time after completion of the landfill the groundwater conditions within the landfill will generally reach a quasi-equilibrium and the concentration of contaminant within the landfill will reach a maximum value. The volume of leachate at this time may be represented in terms of the height of leachate H_f which is defined as the volume of leachate per unit plan area of the landfill.

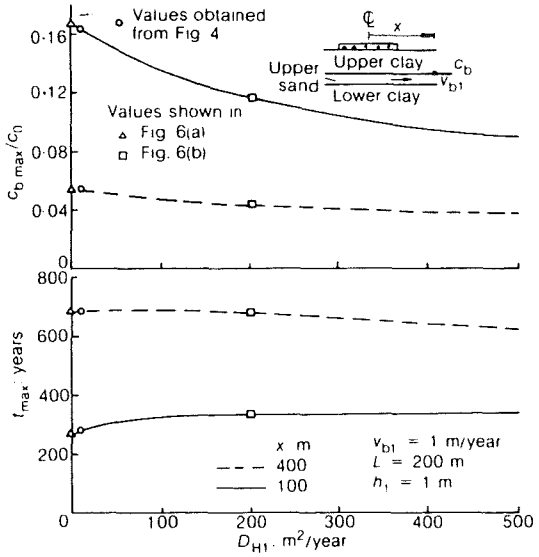

Fig. 5. Variation in maximum concentration $c_{b\,max}$ in the upper sand layer with the coefficient of hydrodynamic dispersion D_{H1}

The mass of contaminant is approximately given by the product of the volume of leachate and the maximum concentration c_0. For a given c_0 the greater the value of H_f, the greater will be the mass of contaminant and thus the peak concentration of contaminant at any particular point in the soil will be greater. This can be appreciated by comparing the result given in Fig. 4 for H_f = 1 m with those presented for H_f = 10 m.

If the limit as H_f tends to infinity is taken (i.e. if the concentration in the landfill is assumed to remain constant at c_0 for all time) then the peak concentration at all points in the soil will eventually reach the landfill value of c_0. This is quite unrealistic and illustrates the need to consider reasonable values of the height of leachate H_f.

Figure 3 illustrated that the horizontal coefficient of hydrodynamic dispersion D_{H1} alters the contaminant plume within the aquifer. However, under some circumstances the effect of the uncertainty regarding D_{H1} on the maximum concentration may be quite modest. For example, Fig. 5 shows the maximum base concentration and the time required to reach this maximum as a function of D_{H1} for v_b = 1 m/year. As can be seen, the effect of D_{H1} on the maximum concentration is greater at the edge of the landfill than at a point 300 m from the landfill. For this particular value of v_b increasing D_{H1} reduces the magnitude of the maximum concentration and increases the time required to reach this maximum. However, this is not always the case. For other values of v_b and other geometries, different trends are observed and the effect of D_H should be examined for each specific case.

If the maximum concentration at a point in the soil mass is considered it is found that the magni-

tude of this maximum value $c_{b\,max}$ (and the time t_{max} required to reach this maximum) will depend on the horizontal advective velocity v_{b1} in the upper aquifer. For example, Fig. 6 shows the variation in this maximum value at two points in the upper aquifer ($x = 100$ m and $x = 400$ m) as a function of the assumed value of v_{b1}. An inspection of Fig. 6 reveals that the magnitude of the maximum concentration at $x = 100$ m increases for an increasing velocity v_{b1} up to a critical velocity v_{b1} of approximately 0·2 m/year and 0·6 m/year for D_{H1} values of 0·2 m^2/year and 200 m^2/year respectively. The magnitude of the maximum concentration becomes smaller for values of v_{b1} that are in excess of these critical velocities. Looking at the results for $x = 400$ m, a similar but more noticeable trend is observed. In this case the critical velocities are approximately 1·5 m/year and 2 m/year for D_{H1} values of 0·2 m^2/year and 200 m^2/year. At low velocities there is a substantial attenuation of the maximum concentration with increasing distance from the landfill. The difference between the maximum concentration at $x = 100$ m and $x = 400$ m becomes smaller as the velocity v_b increases.

The trends observed in Fig. 6 are the result of two competing mechanisms. Firstly the longer it takes for the contaminant to move along the aquifer to the point of interest the more diffusion can occur into the adjacent clayey layers and hence the lower the concentration. The time interval between when the contaminant is released into the aquifer and when it arrives at a point outside the landfill is primarily dependent on the advective velocity v_{b1}. Thus the lower the value of v_b, the more attenuation can occur before the contaminant reaches point x. Beneath the landfill, diffusion out of the aquifer can only be down into the underlying clayey layer (at least until the maximum concentration is reached). However, at points outside the landfill diffusion can occur both upwards and downwards from the aquifer into the two adjacent clayey layers and this gives rise to the rapid attenuation in maximum concentration with distance away from the landfill.

The second mechanism is that of dilution of concentration due to mixing with water in the aquifer. The greater the velocity v_{b1} the greater will be the actual dilution of contaminant. Thus it can be seen that reducing the velocity v_{b1} will decrease the concentration by allowing more diffusion to occur, while increasing the velocity v_{b1}

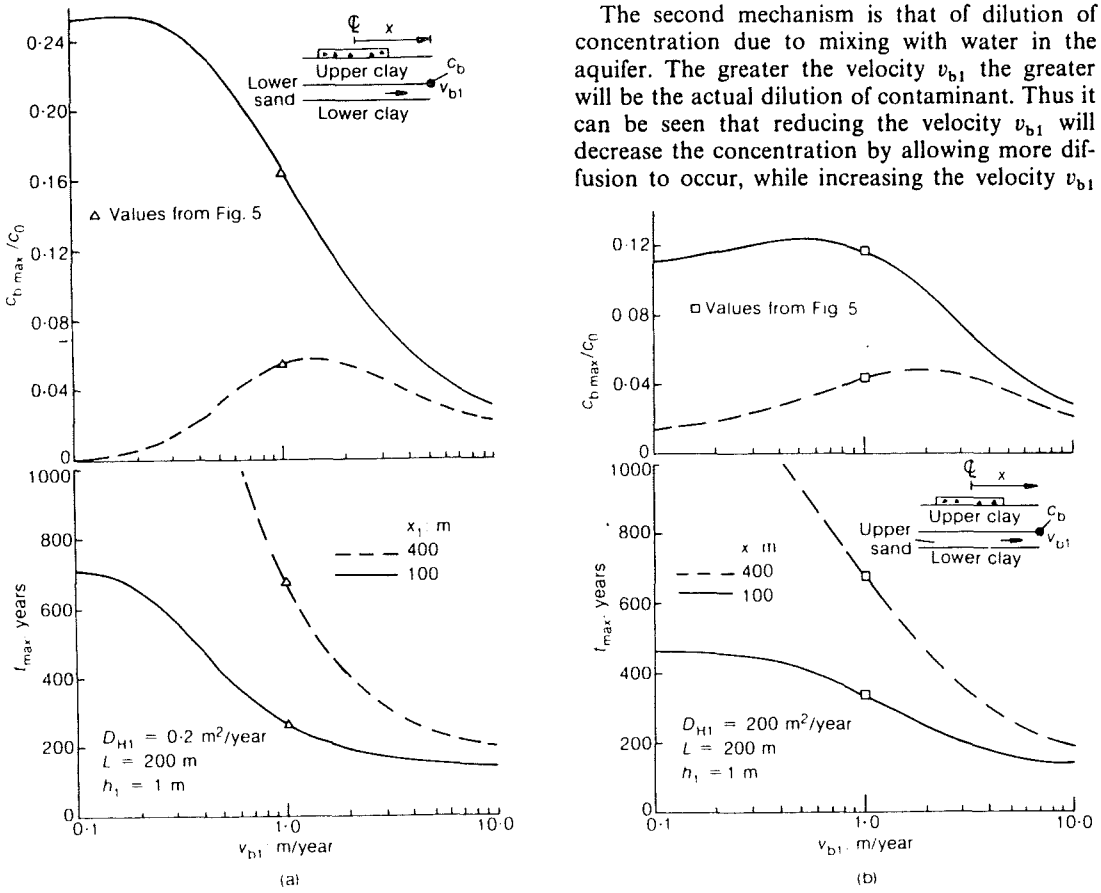

Fig. 6. Variation in maximum concentration $c_{b\,max}$ in the upper sand layer with advective velocity v_{b1}: (a) D_{H1} = 0·2 m^2/year; (b) D_{H1} = 200 m^2/year

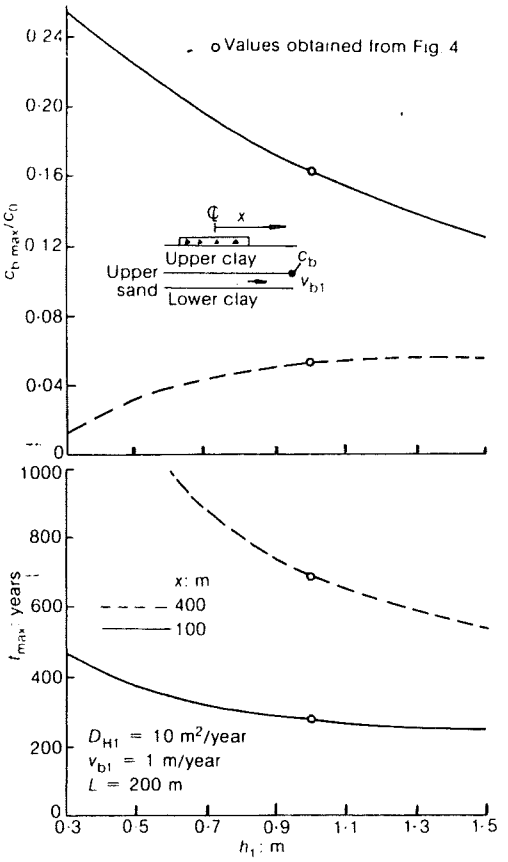

Fig. 7. Variation in maximum concentration $c_{b\,max}$ in the upper sand layer with layer thickness h_1

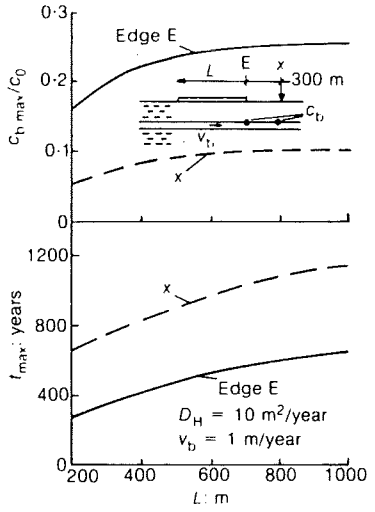

Fig. 8. Variation in maximum concentration $c_{b\,max}$ in the upper sand layer with landfill width L

will decrease the concentration by allowing more dilution. Because of these conflicting mechanisms, there is a critical velocity which results in the greatest peak concentration at a particular point.

For a given problem, the critical velocity will depend on the position x of interest and D_{H1}. This has important practical consequences since it implies that it is not necessarily conservative to design for either the maximum or the minimum expected velocity in the aquifer.

The thickness of the aquifer will have an effect on the concentration of contaminant at various locations along the aquifer. As shown in Fig. 7, the concentration of contaminant at the edge of the landfill (x = 100 m) tends to decrease as the thickness of the aquifer is increased. This is primarily because of an increased dilution of contaminant which occurs for large h (all other things being equal) due to the correspondingly higher flow in the aquifer. At points well away from the landfill (e.g. x = 400 m) the concentration of contaminant may increase with increasing thickness of aquifer (again, all other things being equal) because the relative diffusion into the adjacent clayey soil is reduced. From a consideration of the result given in Figs 6 and 7 it can be seen that there is a fairly complex interaction between the effects of aquifer thickness, advective velocity in the aquifer and the distance to the point of interest on the maximum concentration expected to occur at that point.

All the foregoing results have been for a landfill 200 m wide (i.e. L = 200 m). For the other basic parameters given in Table 1, Fig. 8 shows the variation in the maximum concentration at the edge and 300 m from the edge of a landfill of variable width L. For the problem considered it can be seen that increasing the width of the landfill increases the concentration at both points of interest although the maximum concentration tends to become asymptotic to a constant value for L approaching 1000 m. The increase in concentration with L arises because of the increased mass loading of the aquifer which arises from a large total mass of contaminant within the landfill. The tendency of the asymptote to reach a constant value for very large L arises because significant diffusion can occur into the underlying clay between the time that the contaminant enters the aquifer near the upstream edge and the time that it approaches the downstream edge when the width of the landfill, L, is large. (The value of L at which this occurs will depend on v_{b1}, h, H_f and the other parameters.)

CONCLUSION

A technique for the analysis of two-dimensional and three-dimensional pollutant

migration through a layered soil medium has been described. This formulation permits a consideration of the depletion of contaminant in the landfill with time as well as the effects of diffusion, dispersion and advection in a system involving one or more confined aquifers.

To illustrate some potential applications for the theory, consideration was given to the concentration of contaminant within a confined aquifer where attenuation could occur due to diffusion into the clayey soil above and below the aquifer. Particular attention was focused on the effects of the horizontal advection velocity and the coefficient of hydrodynamic dispersion within the aquifer together with the thickness of the aquifer. It was shown that there is a fairly complex interaction between the effect of these parameters on the maximum concentration reached at a point and it is not necessarily conservative to consider only the bounding values of the expected range of parameters when attempting to ascertain the maximum effect that the landfill will have on the water quality within the aquifer. Finally, the effect of landfill size on the concentration of particular points in the aquifer was examined.

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1-D Pollutant Migration in Soils of Finite Depth

By R. Kerry Rowe,[1] M. ASCE and John R. Booker[2]

Abstract: A technique for the analysis of 1-D pollutant migration through a clay layer of finite depth is presented. This formulation includes dispersive and advective transport in the clay as well as geochemical reactions and permits consideration of the depletion of contaminant in the landfill with time as well as the effect of ground-water flow in a permeable stratum beneath the clay layer. A limited parametric study is presented to illustrate the effect of considering these factors in the analysis. It is shown that for most practical situations the concentration of contaminant within the ground water beneath the landfill will reach a peak value at a specific time and will then decrease with subsequent time. It is shown that the magnitude of this peak concentration and the time required for it to occur are highly dependent upon the mass of contaminant within the landfill and the sorption capacity of the clay. Other important factors which are examined include the thickness of the clay layer, the advection velocity (relative to the dispersivity), and the ground-water flow velocity in any permeable strata beneath the clay layer. The implications of these results for optimizing the design of clay liners is then discussed.

Introduction

Geotechnical engineers are becoming increasingly involved with the problems of pollutant migration through soil. This involvement arises from concern regarding the contamination of the ground-water system by toxic substances which have been, or are being, stored in landfills and lagoons. Often, these disposal sites are located in a clay deposit or have a clay liner. Since the movement of pollutant through relatively impermeable soils is quite slow, the time required for severe contamination of the surrounding ground water may range from several to hundreds of years. The design of these disposal sites should, and now generally does, require consideration of the likely contamination of the surrounding ground-water system in both the short and long term.

Optimal design of waste-disposal facilities requires an understanding of the fundamental mechanisms and the material properties in the appropriate chemical and hydraulic environment, as well as the availability of mathematical models which can be used to make quantitative predictions of liner performance. This paper is concerned with the latter of these requirements.

The key factors governing contaminant migration are advection, dispersion, and chemical reaction where, for many applications involving clay liners or clay deposits, the advective transport is small compared with that due to dispersion (e.g., Refs. 4, 9 and others). Several inves-

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tigators have examined this general problem using both analytical and numerical techniques (1,5). However, existing solutions do not appear to encompass a number of important practical situations.

In this paper, a technique will be developed for the analysis of one-dimensional contaminant transport of a single solute in a layer of finite thickness. While various aspects of the problem examined herein have been recognized in the literature, the present approach differs from previous work in that it considers the combination of the effects of advection, diffusion-dispersion, and chemical retardation in one model with a finite quantity of pollutant in the landfill and moving ground-water beneath the clay deposit-liner. The effect of considering these different factors is demonstrated by means of a parametric study and the implications for design are discussed. Problems involving two-dimensional flow, layering, and multiple interacting solute transport will be examined in subsequent publications.

Theory

Governing Equations.—Assuming that the physical processes of molecular diffusion and mechanical dispersion can both be treated as Fickian type spreading mechanisms, the two effects can be lumped together in terms of a single "coefficient of hydrodynamic dispersion" D. It is found that for one-dimensional transport in the z direction, the mass flux f is given by

$$f = nvc - nD\frac{\partial c}{\partial z} \quad \ldots\ldots (1)$$

in which the first and second terms on the right-hand side represent the advective and diffusive-dispersive transport, respectively; n = the effective porosity; c = the concentration (mass per unit volume of fluid); and v = the average linearized seepage velocity. The velocity $v_a = nv$ will be used to describe advection when analyzing the results. The quantity v_a is variously called the advective velocity, the superficial velocity, the apparent velocity, the Darcy velocity, or the discharge velocity. For the sake of uniformity and simplicity of presentation, the term advective velocity will be used throughout this paper.

Consideration of mass balance gives,

$$-\frac{\partial f}{\partial z} = n\frac{\partial c}{\partial t} + g(c,t) \quad \ldots\ldots (2)$$

in which the function $g(c,t)$ describes the geochemical reaction. For equilibrium controlled ion exchange, where the concentration of one exchange ion is relatively low, the absorption of this species can often be approximated by a linear relationship between the contaminant absorbed and the concentration in the pore fluid, and therefore

$$g(c,t) = \rho K\frac{\partial c}{\partial t} \quad \ldots\ldots (3)$$

in which ρ = the bulk density of solid; and K = the distribution coefficient which may be determined experimentally for a given soil and solute species.

The linear relationship implied by Eq. 3 is a simplification of the real situation; however, provided that the parameter K is determined over a representative range of concentrations, this approach can be shown to be adequate for many practical applications (the use of this linear approximation is analogous to the use of secant elastic moduli in settlement predictions). The distribution coefficient K may be estimated from backanalysis of the results of laboratory column tests (21) or using batch techniques (e.g., 11, 12, 14).

Combining Eqs. 2 and 3 gives the equation

$$(n + \rho K)\frac{\partial c}{\partial t} = -\frac{\partial f}{\partial z} \quad \ldots\ldots (4)$$

The equations developed in this paper are for saturated flow. However, it is likely that the approach could also be approximately used for situations where there is advective-dispersive transport through partially saturated soil provided that the relevant parameters were determined under conditions similar to that anticipated in the field.

Boundary Conditions.—Landfills are of finite extent and have a limited active life. Typically, landfills are constructed in cells. Decomposition will commence immediately after construction of a particular cell; for a period of time (which will depend upon the specific conditions) the concentration of a particular contaminant in the leachate will increase until a maximum is reached. This process may take several years and can be modeled, using superposition, if the details are known. However, if the landfill is constructed on a clay deposit or clay liner, the time to reach peak concentrations is often small compared with the time scale imposed by the slow pollutant migration through the clay. Thus, in many practical situations, it can be assumed that the most hazardous pollutant has a maximum concentration, c_o, shortly after construction (time zero) and that this concentration will then decrease with time as leachate is transmitted through the soil. If the equivalent height of leachate in the landfill is H_f (the equivalent height of leachate equals the volume of leachate divided by the plan area of the landfill), then the surface concentration of any time t (after the peak concentration is reached) is given by

$$c(t) = c_o - \frac{1}{H_f}\int_0^t f_o(c,\tau)\,d\tau \quad \ldots\ldots (5)$$

in which $f_o(c,\tau)$ is the surface flux at $z = 0$ and is defined by Eq. 1.

The height of leachate H_f will probably vary somewhat, due to seasonal fluctuations. However, since these changes are rapid compared with the time scale of the problem, it is reasonable to use an average value of H_f in Eq. 5. The case most frequently investigated by previous workers (e.g., Refs. 3, 13, 16, 17, 19) is that where the surface concentration remains constant (i.e., is independent of time). It follows from Eq. 5 that this can only occur if H_f is very large (denoted in this paper by $H_f = \infty$). This assumption of a constant surface concentration is extremely conservative and, as will become apparent in the following sections, masks many important aspects of the problem of pollutant migration through a finite layer.

If the clay liner (which may be natural or manmade) is underlain at some finite depth H by a far more permeable stratum with ground-water flow in the horizontal direction at an apparent (superficial, Darcy) velocity v_b, then solute will be transported away from the landfill at a rate dependent on the velocity, porosity and geometric dimensions of this layer. The concentration in the underlying more permeable stratum will only tend to zero when the base velocity is sufficiently large to remove the discharged leachate. However, in many cases the velocity will be relatively small and there will be a change in concentration with time in this stratum. To provide a means of estimating this concentration, it will be assumed that the concentration $c_b(t)$ in the underlying permeable stratum does not vary with vertical or horizontal position and that solute transport in this layer is only by advection (this is not strictly true and c_b really represents an average value beneath the landfill. This approximation is considered to be adequate for many practical situations; the more general two-dimensional case where the concentration varies with lateral position beneath the landfill will be considered in a subsequent article).

Consider a landfill with geometry as indicated in Fig. 1. The net change in the mass m of solute in the saturated permeable stratum beneath the landfill at any time t will be equal to the difference between the flux into the permeable stratum from the landfill and the flux from beneath the landfill due to flow in this stratum given by

$$m = \int_0^t WL\, f_b(c,\tau)\, d\tau - \int_0^t Wh\, v_b c\, d\tau \quad \ldots\ldots (6)$$

in which $f_b(c,\tau)$ = the flux into the permeable base stratum at $z = H$ and is given by Eq. 1; and W and L = the width and length of the landfill, respectively (the length is the average dimension of the landfill in the direction parallel to the velocity in the underlying permeable layer). It is implicitly assumed here that the permeable stratum is confined by a lower impermeable boundary and so the volume of water in this permeable stratum, beneath the landfill, depends on its thickness, h (thus, the assumption that c_b is independent of position is likely to be most appropriate when h is no more than a few meters). It follows from Eq. 6 that the corresponding base concentration c at time t is given by

$$c = \int_0^t \frac{f(c,\tau)}{n_b h}\, d\tau - \int_0^t \frac{v_b c}{n_b L}\, d\tau; \quad z = H \quad \ldots\ldots (7)$$

General Solution.—If we introduce a Laplace transform

$$\bar{c} = \int_0^\infty \exp(-st)c(t)dt \quad \ldots\ldots (8)$$

into Eqs. 4, 5 and 7, the governing Eq. 4 reduces to

$$s\bar{c} = -E\frac{\partial \bar{f}}{\partial z} \quad \ldots\ldots (9a)$$

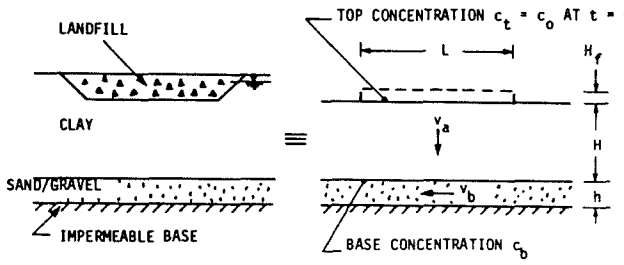

FIG. 1.—Problem Description

in which $\bar{f} = n\left(\bar{c}v - D\frac{\partial \bar{c}}{\partial z}\right)$ ..... (9b)

and $E = \frac{1}{n + \rho K}$ ..... (9c)

which must be solved subject to the transformed boundary conditions:

$$\bar{c} = \frac{c_o}{s} - \frac{\bar{f}}{sH_f} \quad \text{at} \quad z = 0 \quad \ldots\ldots (10)$$

$$\bar{c} = \frac{\bar{f}}{sn_b h} - \frac{v_b \bar{c}}{sn_b L} \quad \text{at} \quad z = H \quad \ldots\ldots (11)$$

Considering solutions of the form

$$\bar{c} = A \exp(kz) \quad \ldots\ldots (12a)$$

$$\bar{f} = B \exp(kz) \quad \ldots\ldots (12b)$$

then from Eq. 9,

$$\begin{bmatrix} s & kE \\ n(v - kD) & -1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0 \quad \ldots\ldots (13)$$

and since the determinant must be zero,

$$k^2 - \frac{v}{D}k - \frac{s}{nED} = 0 \quad \ldots\ldots (14)$$

This equation has roots α, β

$$\alpha = \frac{v}{2D} + \sqrt{\left(\frac{v^2}{4D^2} + \frac{s}{nED}\right)} \quad \ldots\ldots (15a)$$

$$\beta = \frac{v}{2D} - \sqrt{\left(\frac{v^2}{4D^2} + \frac{s}{nED}\right)} \quad \ldots\ldots (15b)$$

Thus $\bar{c} = P \exp(\alpha z) + Q \exp(\beta z)$ ..... (16a)

and $\bar{f} = nD\beta P \exp(\alpha z) + nD\alpha Q \exp(\beta z)$ ..... (16b)

Substituting Eq. 16 into Eqs. 10 and 11 and evaluating at $z = 0$ and $z = H$, respectively, gives

$$\begin{bmatrix} \left(1 + \frac{nD\beta}{sH_f}\right) & \left(1 + \frac{nD\alpha}{sH_f}\right) \\ \left(1 - \frac{nD\beta}{sn_bh} + \frac{v_b}{sn_bL}\right) \exp(\alpha H) & \left(1 - \frac{nD\alpha}{sn_bh} + \frac{v_b}{sn_bL}\right) \exp(\beta H) \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix}$$

$$= \begin{bmatrix} \frac{c_o}{s} \\ 0 \end{bmatrix} \quad \text{(17)}$$

which can be solved for the constants P and Q.

Eqs. 15–17 define the solution at any depth z in transformed space. This solution can be numerically inverted for any time t using the technique proposed by Talbot (24).

These equations can be inverted analytically for a number of limiting cases as shown in Appendix I.

Results

The parameters governing the migration of pollutants through clayey soils and clay liners vary significantly depending upon the nature of any particular site and species of pollutant. Although the theory is amenable to considering the entire spectrum of possible combinations of parameters, it is not possible to do so in this article. Thus, to illustrate the significant factors arising from the use of this formulation, consideration will be given to the hypothetical case of a 200 m long (see Fig. 1) landfill resting on a homogeneous clay stratum with porosity $n = 0.4$ underlain by a relatively permeable, 1 m thick, sand stratum ($h = 1$ m) with porosity $n_b = 0.3$. The dispersion coefficient will be taken to be $D = 0.01$ m^2/y. Unless otherwise noted, the clay layer is assumed to be 2 m thick ($H = 2$ m), the apparent advection velocity, v_a, is zero and there is no geochemical reaction ($\rho K = 0$).

Effect of Leachate Height and Base Velocity.—The total mass of any particular species of pollutant available for migration from the landfill is directly related to the concentration of that species within the leachate and to the total volume of leachate. The concentration can be measured. It is more difficult to determine the volume of contaminant in the landfill. However, it is possible to make estimates of likely upper and lower bounds for the volume of leachate by considering the porosity of the landfill material and the height of the watertable within the landfill. The height of leachate H_f represents the volume of leachate divided by the area of the landfill. An examination of a limited number of landfills would suggest that H_f is likely to range from 0.5–10 m, with values of 1–5 m being most probable (based on 8, 15, and other unpublished reports). H_f is related to the water content of the landfill material and, with the exception of lagoons, will be less than the total height of fill material.

Recognizing that contaminant will eventually diffuse through clay liners into any underlying ground water (even if there is no advection), the design and certification of landfills should involve consideration of

the maximum expected concentration of contaminant within the ground water. Often, the initial concentration of pollutant within the leachate will be up to several orders of magnitude higher than recommended levels for potable waters and the designer will be concerned with maintaining quite low concentrations at the base of the liner for a considerable time after emplacement, e.g., the leachate concentrations of chloride and phenol in a municipal landfill at Sarnia, Ontario (Crooks and Quigley, personal communication) are 3,000 mg/L and 0.150 mg/L, respectively whereas recommended limits for these in drinking water are 250 mg/L and 0.001 mg/L, respectively (25).

The effect of the height of leachate H_f upon the surface concentrations for the limiting cases where the base concentration c_b is zero (i.e., the base velocity approaches infinity) and the base velocity is zero are shown in Figs. 2(*a* and *b*), respectively. With $c_b = 0$ (infinite base velocity), the surface concentration decreases fastest for low values of H_f but in all cases tends to zero in the steady state. In the second limiting case, with zero base velocity, the surface concentration decreases until it reaches a steady-state value (which depends upon H_f), and the base concentration increases monotonically with time until it attains a steady-state value equal to the surface concentration.

The situation where there is zero base velocity provides the worst-case condition for a given value of H_f. In most practical situations where a landfill is underlain by a permeable stratum, there will be some average, nonzero base velocity v_b. From the results given in Fig. 2, it may be anticipated that for this case the base concentration will increase with time reaching a maximum value at some time t_{max} and then will decrease until the steady state of zero concentration is reached. This behavior is shown in Fig. 3 for $H_f = 1$ m and a number of apparent base velocities v_b.

Similar trends are observed for all values of H_f except for the situation where H_f tends to infinity. This limiting case corresponds to a constant surface concentration and is fundamentally different from that for any finite value of H_f. Here, the base concentration increases with time and becomes asymptotic to a steady-state value which depends on the base velocity. This case represents the upper limit in terms of both the maximum base concentration and the time required to reach the maximum value. This situation is unlikely to occur in practice.

Figs. 4(*a–b*) summarize the effect of base velocity upon the maximum base concentration and the time required to achieve this maximum for a range of values of H_f. As might be expected, increasing the base velocity results in greater dilution of contaminant; thus, the peak concentration is lowered and is reached at an earlier time. For moderate to large velocities (i.e., $v_b > 10$ m/y) the maximum base concentration varies inversely with the base velocity whereas the time required to reach this maximum is relatively independent of velocity. This observation allows convenient extrapolation for velocities greater than those shown on Fig. 4.

In summary, it is noted that for a finite mass of contaminant and nonzero base velocity, the concentration of pollutant in the ground water beneath the liner will reach a maximum value c_b at some definite time t_{max} and will decrease for greater times. This maximum value can be used in design to ensure that contamination of the ground water never exceeds a specified level. In this context, the design of the liner can be

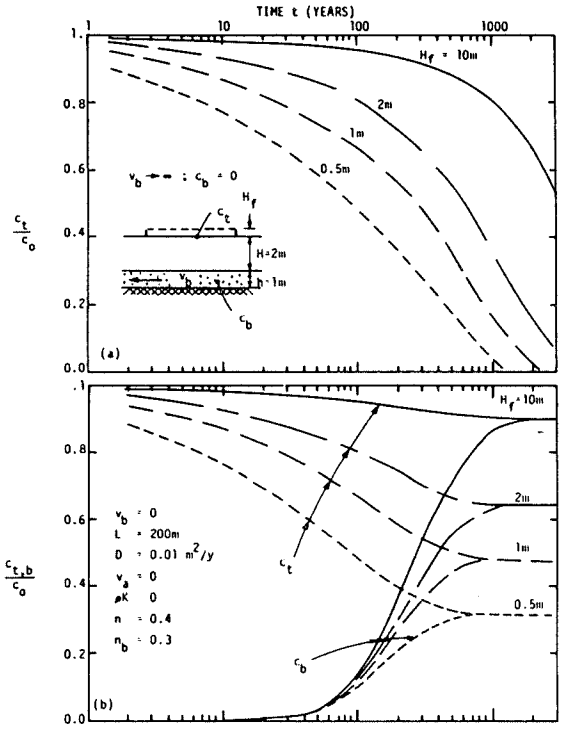

FIG. 2.—Variation in Concentration With Time for Different Heights of Leachate H_f: (*a*) Base Concentration Zero ($v_b \to \infty$); (*b*) Base Velocity Zero ($v_b = 0$)

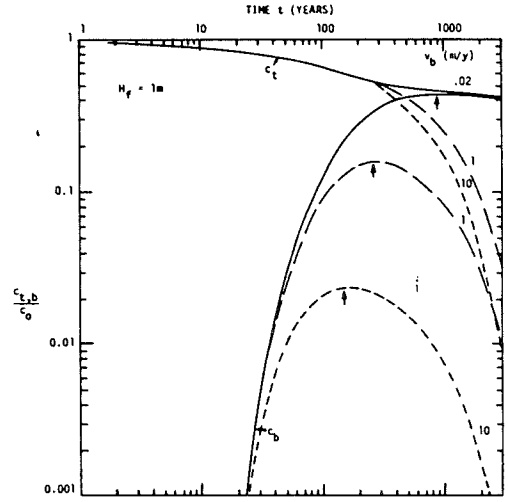

FIG. 3.—Variation in Top and Base Concentrations With Time for Different Base Velocities v_b: H_f = 1 m

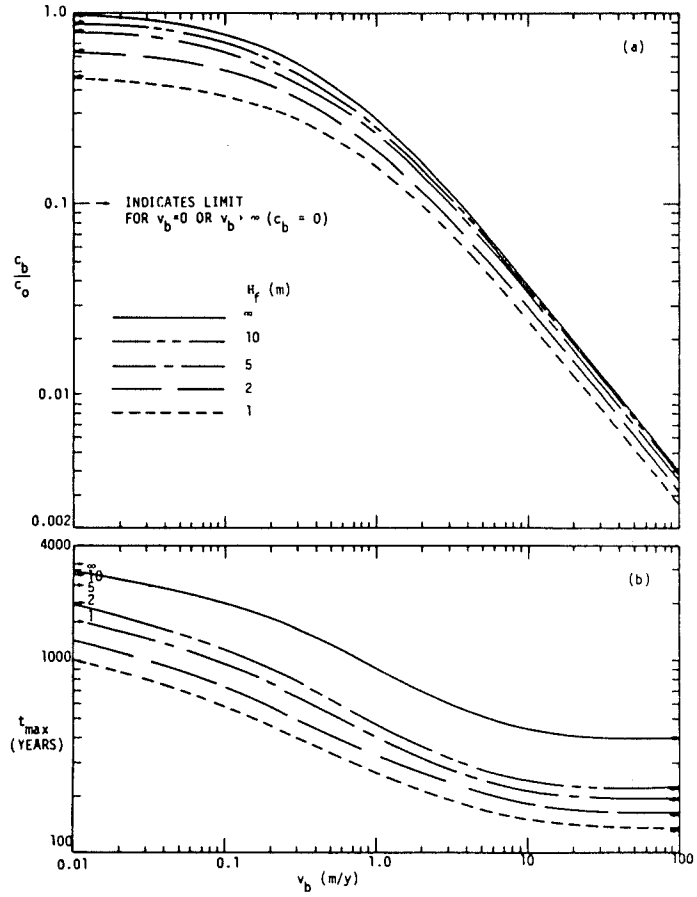

FIG. 4.—Effect of Base Velocity v_b Upon: (*a*) Maximum Base Concentration c_b; (*b*) Time Required to Reach Maximum Concentration, t_{max}

optimized by considering a realistic value of leachate height and base velocity, rather than highly conservative limiting values.

Effect of Geochemical Reaction.—The absorption process is most easily represented in terms of the dimensionless quantity ρK. The value of ρK will depend upon the soil properties, chemical reactions and their rates, and the range of concentration. Typical values lie between 0 and 100 although much higher values have been reported (e.g., see 6, 7, 10–12, 14, 22).

If, as is usually assumed, the surface concentration is constant ($H_f = \infty$) then the effect of the geochemical reaction is simply to slow down the dispersion-advection process. Thus, the maximum base concentration is independent of the geochemical reaction although the time t_{max} required to reach this maximum does increase significantly. This is not true when H_f is finite.

In the previous section, it was shown that the effect of the height of

leachate being finite was to reduce the maximum base concentration. This effect is greatly increased by any geochemical reaction. Fig. 5 shows the variation in both the top and bottom concentration with time for the case where there is no sorption ($\rho K = 0$) and where there is moderate sorption ($\rho K = 10$). In both cases, the base concentration increases with time until it reaches a maximum value and then decreases. As might be expected, the geochemical reaction increases the time required to reach the peak concentration by approximately one order of magnitude for $\rho K = 10$. Of greater interest, in comparison with the situation for $H_f = \infty$, is the fact that the maximum base concentration is also reduced by approximately one order of magnitude.

At small to moderate times, the sorption process results in lower surface concentrations for $\rho K = 10$ than for $\rho K = 0$. At times greater than that required to reach the maximum concentration for $\rho K = 10$, the surface concentration for $\rho K = 10$ exceeds that for lower values of ρK. The reason for this is that the clay acts as a buffer. At small to moderate times, the clay absorbs pollutant from the pore fluid thereby reducing both the concentration and the concentration gradient. Each position in the clay will reach a maximum concentration at some point in time. Once the time has been reached, the clay will cease sorbing pollutant. Depending on the details of the particular sorption process involved, the contaminant sorbed onto the clay will either remain fixed to the clay or in the worst case, as assumed here, will desorb back into the pore fluid. It is this release which is responsible for the higher values of concentration throughout the deposit for $\rho K = 10$ than for $\rho K = 0$ at large times. This buffering role of the clay only becomes apparent if the finite mass

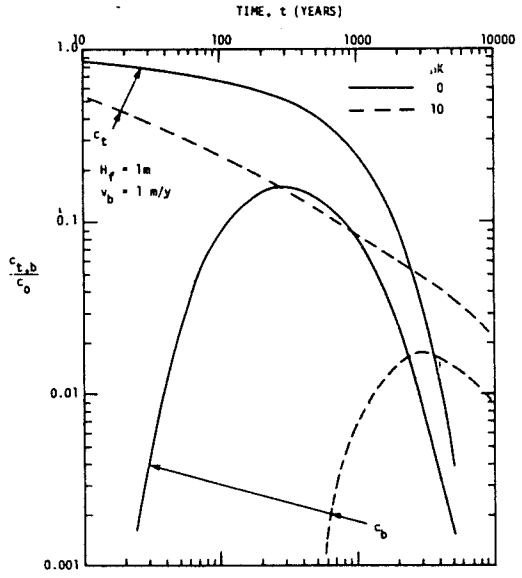

FIG. 5.—Effect of Geochemical Reaction Upon Variation of Top and Base Concentration With Time: H_f = 1 m

of pollutant in the landfill is considered.

Figs. 6 and 7 show the effect of geochemical reaction upon the maximum base concentration and the time required to reach this maximum for v_b = 1 m/y. For $\rho K < 0.1$, the geochemical reaction has negligible

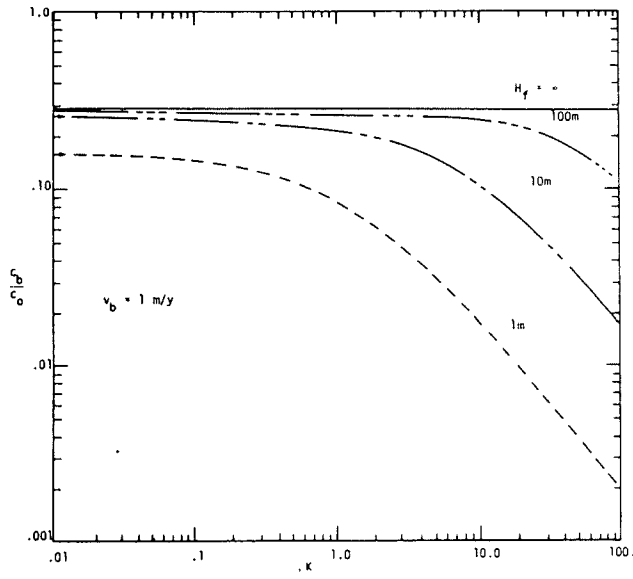

FIG. 6.—Effect of Sorption Upon Maximum Base Concentration: v_b = 1 m/y

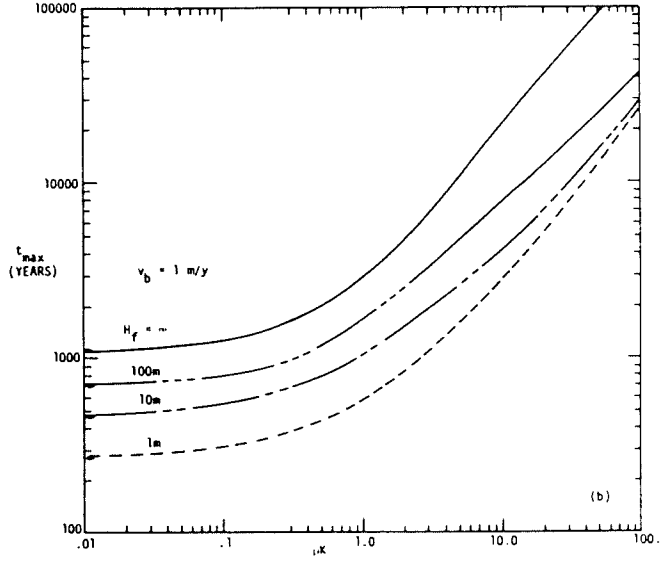

FIG. 7.—Effect of Sorption Upon Time to Reach Maximum Concentration: v_b = 1 m/y

effect upon the base concentration. If $\rho K > 1.0$, then the geochemical reaction significantly decreases the base concentration for most practical cases (i.e., $H_f \leq 10$ m) and should be considered in design. The effect of the geochemical reaction is most pronounced for small heights of leachate (H_f) but may still warrant consideration for quite large heights ($H_f < 100$ m) if the geochemical reaction is strong (i.e., $\rho K > 10$). For high values of ρK, the base concentration varies inversely with ρK.

The time required to reach the maximum base concentration (see Fig. 7) increases for $\rho K > 1.0$ and approaches a linear relationship for high values of ρK. Also of interest is the fact that the relationship between

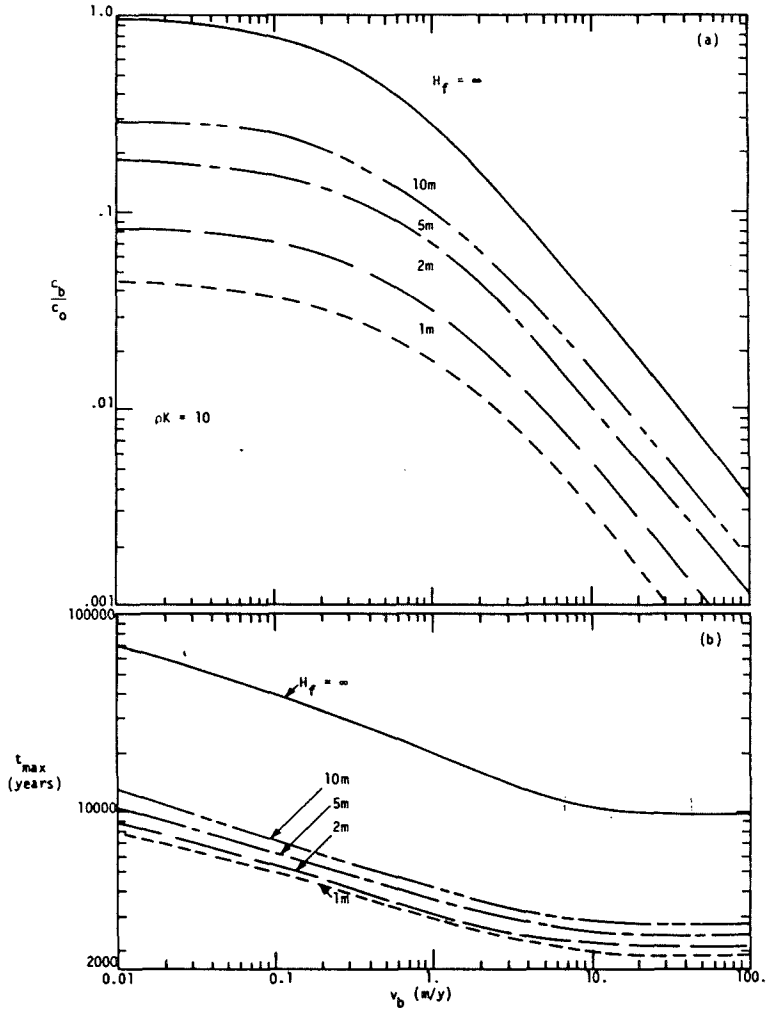

FIG. 8.—Effect of Base Velocity v_b Upon: (*a*) Maximum Base Concentration c_b; (*b*) Time Required to Reach Maximum Concentration t_{max}: $\rho K = 10$

t_{max} and ρK tends to a single curve for $H_f < \infty$ at very large values of ρK.

The effect of base velocity v_b is shown in Fig. 8 for $\rho K = 10$ and may be compared with the results given in Fig. 4 for $\rho K = 0$. The same general trends are evident in both figures. The effect of base velocity upon base concentration can generally be neglected (to an accuracy of better than 20%) for $v_b < 0.1$ m/y although it should be noted even these small base velocities may reduce the value of t_{max} by up to a factor of 2. For $v_b > 10$ m/y, the relationship between base concentration and base velocity tends to the inverse relation whereas time t_{max} is relatively independent of v_b. Of particular importance is the fact that the geochemical reaction greatly enhances the effect of H_f. For $\rho K = 0$, the value of c_b varies by up to a factor of 2.2 for H_f in the range $1 \leq H_f \leq \infty$. For $\rho K = 10$, the corresponding variation in c_b is up to a factor of 22.

Effect of Layer Thickness.—In design, the thickness and known attenuation potential (ρK) of the clay layer isolating the landfill from the underlying ground-water system may be used to control the maximum base concentration (e.g., Ref. 2). Figs. 9–12 show the variation in c_b and t_{max} with clay liner thickness H for $\rho K = 0$ and $\rho K = 10$. Increasing the layer thickness substantially reduces the maximum concentration ever reached at the base and quite obviously increases the time required to reach this maximum. For example, with $\rho K = 0$, increasing the clay liner thickness from 0.5 m to 4 m decreases the $c_{b(max)}$ by up to an order of magnitude (see Fig. 13) and increases the time t_{max} by up to an order of magnitude.

The effect of layer thickness is greatly increased by consideration of the height of leachate and the geochemical reaction. Thus, the thickness of liner required to ensure that a specified maximum base concentration is never exceeded may be significantly reduced by considering the interaction of all of these factors.

Effect of Advection Velocity.—The advection velocity v_a within the clay deposit will depend on the hydraulic conductivity of the clay and the difference between the head in the landfill and the head in the underlying aquifer. For the 1-D conditions assumed in this paper, v_a can be readily calculated from a knowledge of these quantities. If, as is often the case, the head in the landfill is greater than in the aquifer, the velocity v_a is positive and the theory used in this paper can be used directly. If the head in the landfill is less than in the aquifer, there will be upward flow into the landfill. With minor modifications to the boundary conditions, this case can also be examined. It should be noted that even with upward flow, diffusion through the clay liners may still occur although clearly, the rate of mass transfer will be much smaller than when there is downward flow.

All the previous results were obtained for the limiting case of zero advection. Whenever possible, a disposal site should be located such that there is very little seepage through the waste into the surrounding soil and local ground-water system. If the disposal site must be located in a region where there is likely to be significant seepage, then the seepage through the waste may be minimized by containing the waste within a clay liner which is itself fully surrounded by a far more permeable graded filter which effectively creates an equipotential surface around the liner (e.g., see Ref. 18).

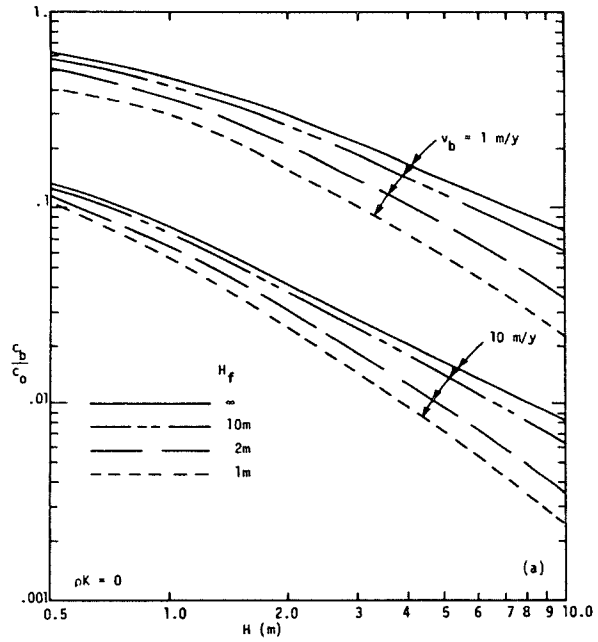

FIG. 9.—Effect of Layer Thickness Upon Maximum Base Concentration: $\rho K = 0$

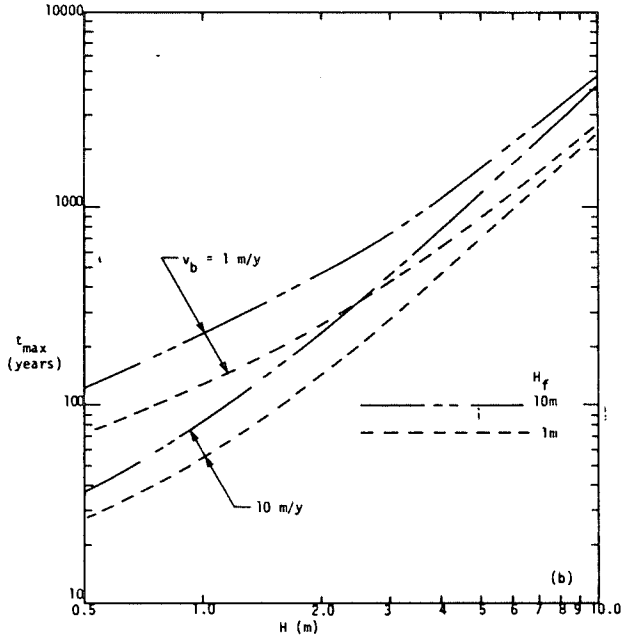

FIG. 10.—Effect of Layer Thickness Upon Time to Reach Maximum Concentration: $\rho K = 0$

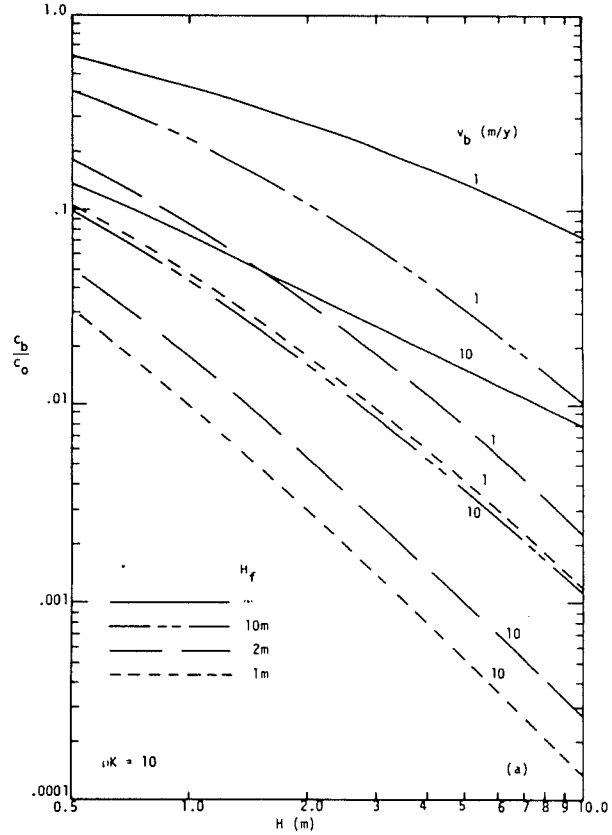

FIG. 11.—Effect of Layer Thickness Upon Maximum Base Concentration, $\rho K = 10$

Although seepage may be minimized, it will never be precisely zero and it is of interest to examine the effect of the apparent (superficial) advective velocity through the liner upon the maximum base concentration. Analyses were performed for a range of advection velocities. The results exhibited the same trends with regard to the effect of base velocity, layer thickness, leachate height, and geochemical reaction as reported in the previous section. However, the advection velocity did increase the maximum base concentration and decrease the time required to reach this concentration as shown for a 2 m thick liner with $\rho K = 0$ in Fig. 13 (the results for other values of ρK are very similar except that the values of c_b and t_{max} differ in approximately the same ratio as they do for $v_a = 0$ in Figs. 6–8).

From Fig. 13, it can be seen that the advective velocity could be neglected for $v_a < 0.0004$ m/y. For v_a up to 0.001 m/y, advection increases base concentration by up to 33% although t_{max} is almost constant. Advective velocity greater than 0.001 m/y may significantly affect the base concentration, with the values of $v_a = 0.01$ m/y being up to 5 times greater than those for $v_a = 0$. The time t_{max} is somewhat less affected by

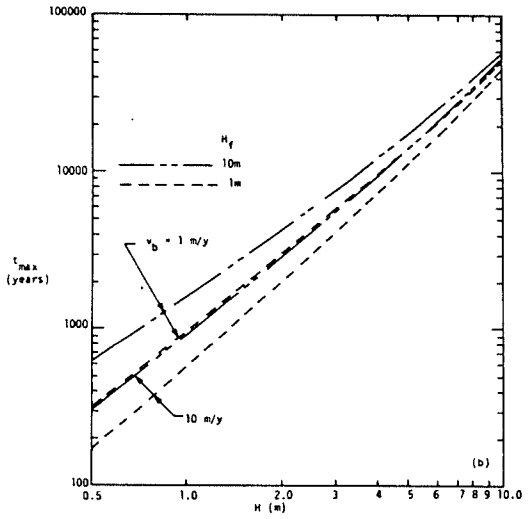

FIG. 12.—Effect of Layer Thickness Upon Time to Reach Maximum Concentration: ρK = 10

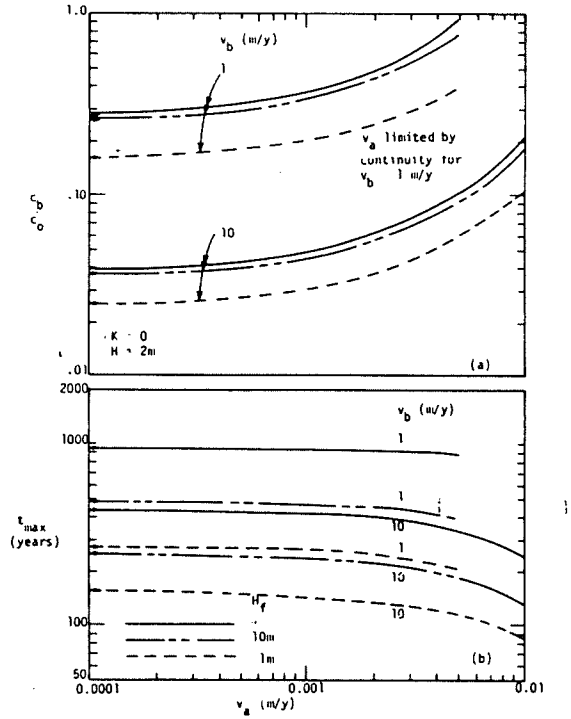

FIG. 13.—Effect of Advection Velocity v_a Upon: (*a*) Maximum Base Concentration c_b; (*b*) Time Required to Reach Maximum Concentration t_{max}

v_a, varying by less than a factor of 2 for the same range in advective velocity. This demonstrates the importance of minimizing the advective velocity in the siting and design of the waste disposal site.

Application

The techniques developed in the theory section can be readily programmed and have been implemented in the program POLLUTE (20). This program involves minimal data preparation and computational effort.

The analysis is suitable for use in situations where the "liner" inhibiting contamination migration is relatively homogeneous and the thickness of the liner is small compared to the dimensions of the landfill (i.e., where H/L is less than 0.05). The analysis can be readily extended to multilayered 2-D and 3-D situations involving single or multiple interacting species and this will be described in subsequent articles. However, it is considered that the simple 1-D case examined here will be adequate for a large range of practical situations. The technique is currently being used in conjunction with a detailed field study of a landfill site at Sarnia, Ontario.

Conclusions

A technique for the analysis of 1-D pollutant migration through a soil layer of finite depth has been presented. This formulation permits consideration of the depletion of contaminant in the landfill with time as well as the effect of a flushing velocity in a permeable stratum beneath the clay layer. The technique requires minimal data preparation, is accurate, and is computationally very efficient compared with alternative techniques such as finite element and finite difference methods (e.g., Ref. 23 and many others).

A limited parametric study was performed to demonstrate some of the important features which arise from considering the finite mass of contaminant and a flushing velocity at the base of the clay liner. For a finite mass of contaminant and the range of parameters considered, the following may be concluded:

1. If the base velocity is greater than zero, then the concentration of pollutant in the ground water beneath the liner will reach a maximum value, c_b, at some time t_{max}, and will decrease for greater times. This maximum value can be used in design to ensure that the contamination of the ground water never exceeds a specified level (if the base velocity is zero then the base concentration reaches a maximum and remains at that value for all subsequent time).
2. The magnitude of the maximum base concentration decreases: (a) with decreasing mass of contaminant in the landfill; and (b) with increasing base velocity.
3. Geomechanical reactions (sorption) can greatly affect the magnitude of the maximum base concentration as well as the time required to reach this maximum. The sorption potential will depend on the species of contaminant being considered and the chemical properties of the clay.

For reactive species, the clay adsorbs pollutant when the concentration is increasing. Once the concentration at a point reaches a maximum value, the sorbed contaminant may remain fixed to the clay or, in the worst case, may be released back into the pore fluid as the concentration drops. This beneficial buffering role for some species of pollutant is only apparent if the finite mass of pollutant in the landfill is considered.

4. The effect of sorption is greatest for low to moderate volumes of leachate (i.e., leachate height less than 10 m). Sorption may be particularly important in designing for hazardous substances such as NH_4 and heavy metals although careful consideration of the chemical properties of the clay is necessary.

5. The maximum base concentration can be decreased by increasing clay liner thickness. However, the design of the liner may be optimized by also considering the effects of leachate height, base velocity and any geochemical reaction.

6. The consideration of advection velocity does not alter the trends described above, but it does increase the maximum base concentration and decrease the time required to reach this concentration.

Acknowledgment

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Appendix I.—Solution of Governing Equations for Limiting Cases

As the volume of the landfill tends to infinity, the surface concentration will tend to a constant value c_o. Also taking the limit as the depth H tends to infinity, Eq. 17 yields

$$P = 0, \quad Q = \frac{c_o}{s}$$

$$\text{thus} \quad \bar{c} = \frac{c_o}{s} \exp(\beta z) \qquad (18a)$$

$$\text{in which} \quad \beta z = \frac{vz}{2D} - a\sqrt{s + b^2} \qquad (18b)$$

$$\text{and} \quad a = \frac{z}{\sqrt{nED}} \qquad (19a)$$

$$b^2 = \frac{nEv^2}{4D} \qquad (19b)$$

Eqs. 18 can be inverted analytically yielding $c = J(z,t)$ in which

$$J(z,t) = \frac{c_o}{2} \exp\left(\frac{vz}{2D}\right)\left\{\exp(-ab)\, Erfc\left(\frac{a}{2\sqrt{t}} - b\sqrt{t}\right) + \exp(ab)\, Erfc\left(\frac{a}{2\sqrt{t}} + b\sqrt{t}\right)\right\} \qquad (20)$$

which corresponds to the classic solution (e.g., Ref. 19).

Considering now the limiting case as the surface concentration tends to a constant c_o but the layer depth H is finite, being underlain by a permeable stratum which is fully flushed such as $c = 0$ at $z = H$. Then

$$P + Q = \frac{c_o}{s} \qquad (21a)$$

$$P \exp(\alpha H) + Q \exp(\beta H) = 0 \qquad (21b)$$

Solving for P and Q and substituting into Eq. 16*a* yields

$$\bar{c} = \frac{c_o}{s}\left[\frac{\exp(\alpha z + \beta H) - \exp(\beta z + \alpha H)}{\exp(\beta H) - \exp(\alpha H)}\right]$$

$$= -\frac{c_o}{s}\left[\frac{\exp(\alpha z + \beta H) - \exp(\beta z + \alpha H)}{\exp(\alpha H)}\right] \sum_{p=0}^{\infty} \exp[(\beta - \alpha)pH] \qquad (22)$$

Therefore

$$\bar{c} = -\frac{c_o}{s}\left\{\sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}[(p+1)H - z]\right) \exp(\beta[2(p+1)H - z)]) - \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}pH\right) \exp(2pH + z)\beta\right\} \qquad (23)$$

Now, as indicated earlier, an equation of the form $\bar{c} = c_o/s \exp(\beta z)$ has the solution $c = J(z,t)$ in which $J(z,t)$ is defined by Eq. 19 for any z, t and thus, Eq. 20 may be inverted to give

$$c = -\left\{\sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}[(p+1)H - z]\right) J[2(p+1)H - z, t] - \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}pH\right) J(2pH + z, t)\right\} \qquad (24)$$

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